WAVE PROPOGATION STUDIES IN FIBER REINFORCED COMPOSITE MATERIALS USING FINITE ELEMENT METHOD

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CERTIFICATE

This is to certify that the present work titled ' **WAVE PROPAGATION STUDIES IN FIBER REINFORCED COMPOSITE MATERIALS USING FINITE ELEMENT METHOD** ' *by Vidyashankar B R has been carried out under our supervision and has not been submitted elsewhere for the award of a degree.*

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CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF SYMBOLS

c	Physical Constant, Speed of Light
x_i	Co-ordinates
t	Time
C_g	Group Velocity
a_r	Coefficients
d	A Nonzero Real Constant
N	Number of Particles
C_{ijkl}	Elastic Stiffness Coefficient
u_i	Generalised Displacements
W	Weighting Functions for Gaussian Integration
R	Residual
w	Weighting Functions in the Residual
D	Domain
$\hat{n}_j$	Unit Vector
A	Area
N_i	Elemental Shape Functions
v	Displacements
J	Jacobian
ψ	Disturbance Displacement
λ	Wavelength
Ω, ω	Circular Frequency
ρ	Mass Density
ζ	Wave Number
ξ, η	Natural Co-ordinates

SYNOPSIS

In the present work two dimensional harmonic wave propagation in fiber reinforced composite materials has been attempted using Finite Element Method (FEM). Concepts of periodic unit cell and Floquet's conditions has been used in the analysis.

Wave propagation analysis is known to give a good indication of static and dynamic behavior of the medium of propagation. Nature of wave propagation in isotropic materials has been extensively studied by many researches and exact results for the same has been obtained. Recently there is increase in the use of composite materials in critical and non critical components like aerospace structures. Understanding the behavior of these materials under different conditions such as impact and fatigue loads becomes important. Non destructive testing techniques like ultrasonic and acoustic emission techniques which are used to study the material characteristics are based on the principles of wave propagation . As conventional techniques are not adequate, study of wave propagation in these materials becomes necessary.

Attempt has been made to study the wave propagation in composite materials to obtain ten lowest eigen frequencies and corresponding eigenvectors as a function of wave length λ .

Two different composite materials (E_f $/E_m$ = 3.275 and 20.1)are studied and their behavior is compared to that an isotropic material. Frequency spectra of these materials for the propagation in the plane of fibers and in the thickness plane have been obtained.The wave patterns corresponding to optical and acoustical branches of the frequency spectrum are investigated.

Results reveal that composite materials are dispersive in nature as compared to isotropic mamterials. The amount of dispersion depends on the wavelength/frequency , direction and the mode (acoustical and optical) of propagation. It has been observed that dispersion increases with the ratio of stifnesses of individual constituents and in the propagation perpendiculer to the fibers.It has been found out that FEM can be effecitively used to study wave propagation in composite materials.

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION :

Static and dynamic analysis of composite materials is gaining much attention these days as these materials are finding wide range of uses in many critical and non critical structures of aerospace structural components. These applications impose severe restrictions on the behavior of composite materials. Especially under conditions of high strain rate and impulse loading (shockwave during impact) their behavior is to be understood in sufficient detail. Non destructive testing (NDT) techniques like Ultrasonics, Acoustic emission and Acousto Ultrasonics are being widely used as experimental tools to study the structure and hence, behavior of materials both qualitatively and quantitatively. Also since there is a relationship between mechanical properties and wave propagation, study of wave propagation gives another method of determining the mechanical properties apart from conventional methods, like static and vibrational properties.

1.2. WAVE PROPAGATION ANALYSIS :

Harmonic wave propagation analysis deals with studying the mechanics of wave propagation like stress wave propagation in materials. This analysis to determine properties such as E, ν,ω has got certain basic advantages over other methods of analysis. Many complexities that are involved in the solution of governing equation of motion can be avoided if the wave solution technique

is applied to such problems. Though modal superposition analysis can be used to obtain free/forced response of continuua they are not often applicable for cases with complex geometries, plates or systems of infinite or semi infinite extent. Sometimes it may be difficult to obtain mode shapes or frequencies for medium with non uniform properties and heterogeneities. A medium of infinite extent has a continuous band of frequencies and hence mode superposition etc., may not be directly applicable. Some of the above difficulties can be circumvented by applying wave propagation technique.

1.3. COMPOSITE MATERIALS :

A composite material consists of two or more materials mixed in suitable proportion so as to obtain desired qualities. Usually the proportion is determined based on the optimal utilization of properties of the constituents. There will be generally two kinds of constituent materials, viz., a reinforcing or the major load bearing material and a bonding matrix material. Composites are generally classified into particulate reinforced composities and fiber reinforced composites depending upon the geometry of the reinforcing material. The high strength and stiffness, high directionality in the elastic properties (anisotropy) and good formability are some of the attractive features of composite materials.

The complex nature of composite material presents many difficulties in analysing composite materials and to understand mechanical behavior due to the heterogeneity at the micro level (when the area of interest is of the order of fiber

dimensions) because of different material properties , interfacial effects between plies, and viscoelastic behavior of the composite. The general anisotropy and coupling of in plane and out of plane stresses / strains introduce lot of complexities in the analysis.

The material has some inherent defects such as fiber breaks, matrix debonding, delamination etc.. These cause adverse effects in the behavior of these materials under different types of loads. Plastic and viscoelastic nature of matrix material cause flow of the material beyond a certain value of stress.These factors have lead to increasing use of wave propagation analysis as a technique to study the structure and behavior of composites under different loading conditions. Standard methods and solutions for isotropic materials cannot be easily extended to the case of composites. Various theories have been proposed to model the elasto dynamic behavior of these materials.

1.4. FINITE ELEMENT METHOD (F E M) :

It is an approximate analytical technique for solving differential equations. Using variational or weak formulation of the differential equation and then discretising the domain suitably a set of algebraic equations are obtained[1]. These equations which are easier than the original differential equation can be conveniently solved using a digital computer. Finite element method has several advantages over the exact solution techniques and other approximate methods. This technique can easily handle complex shaped domains , complicated boundary conditions, discontinuities, nonhomogenieties , nonlinearities in

material etc.. Advent of high speed digital computers has facilitated rapid increase in the range of applications of this technique in solving realistic problems.

From the above discussion it can be inferred that the wave propagation analysis using FEM can be a good technique to characterise the wave propagation composite materials.

1.5. LITERATURE SURVEY :

Literature available in the field of finite element analysis of wave propagation in composites is limited in comparison to that available in vibration and modal superposition studies. Some work is done in wave propagation analysis using finite difference schemes and special matrix methods [2,3]. Kohn et al. [5] used Rayliegh Ritz procedure to solve variational form of wave equation formed over a single periodic cell using Floquet theory and determined dispersion properties and elastic properties of a few composites using plane wave approximation for two and three dimensions. Natural frequencies and mode shapes obtained are comparable to the available exact results.

Dong and Nelson [6] have studied natural vibrations and wave propagation in laminated composite plates using FEM. They have obtained ten lowest natural frequencies and associated modal displacements, along with stress distributions.

Yang and Lee [7] have used Floquet theory and finite difference scheme for modal analysis. Lowest five modes are accurately and efficiently computed.

Nelson and Navi [4,8] have used FEM to study harmonic wave propagation in composite materials and other orthotropic materials

under plane strain condition. Floquet theory and periodic cell approach has been followed. They have shown that this model can represent accurately wavelengths upto half the lattice dimensions. Frequency spectra for various orthotropic and isotropic materials are obtained. The eigenvalue system has been solved for various values of linear wave number ζ.

Golub et al. [9] have solved an eigen value problem obtained using FEM procedure. They considered steady state 2-D wave propagation in problem and used Floquet theory to obtain a Hermitian system of equations which are then converted to real system of equation using the sparse nature of the matrices. Dispersion relations are obtained from the analysis.

Minagawa et al. [10,11] have used finite elements to study harmonic wave propagation in fiber reinforced and layered composite plates. Unit cell approach is followed here based on Floquet theory. Wave patterns corresponding to acoustical and optical branches (in the Brillouin sense) of frequency spectra are obtained for different wave numbers. It has been observed that for wave lengths greater than the periodic dimensions of the medium and for low frequencies, the medium behaves like a homogeneous one.

Datta et al. [12] have used a stiffness method to study wave propagation in composite plates. They have found that frequency spectrum for 0° laminate differs considerably from that of a (0/90/0) cross ply laminate.

Osaki and Kimpara [2] have made an attempt to study defects in unidirectional fiber reinforced composites by elastic wave propagation. They have used a finite difference scheme for

analytical modeling and ultrasonic technique for experiments.By defining a new parameter obtained using FFT analysis of experimental data they have evaluated the defects in the form of fiber breaks.Vector charts have been used to illustrate the effect of fiber breaks (notched defects) on sine wave propagation based on dynamic simulation.

Mal and Barcohen[13] have, in their study of fiber composites used a matrix method for analytical simulation of leaky lamb wave (LLW) experiments.They have calculated phase velocities of Lamb waves of laminates.Reflection and water loading effects have also been considered.For simple cases they have found good agreements whereas for realistic cases discrepancies have been noticed.

Toshiuki Oshima et al.[14] studied the effect of the orientation of point masses on stress waves. Dynamic response of a rectangular cross sectioned beam with regularly arranged point masses under impact loading is investigated. Results are compared for the cases with discrete and continuum models.

Masanori Koshiba et al. [15] have used FEM and an analytical method to study Lamb wave scattering in an elastic plate wave guide. Effects of internal and surface cracks are investigated. They have found out the effect of wedge angle (of the crack) on the reflection coefficient and associated resonance phenomena.

Considerable work was also done to characterize composite materials using NDT techniques. A brief report of the same is provided here.

Rose [16] has underlined the principles of ultrasonic wave propagation in composite material inspection.The effects of wave velocity, dispersion,reflection coefficients and attenuation

characteristics of the material on the wave profile are studied.

Rose et al. [17] attempted to analyze actual wave profile in an unidirectional graphite epoxy plate using through transmission technique. Wave velocities are measured for different fiber orientations.From the wave velocities, Young's modulii in longitudinal and transverse direction are computed. Other engineering constants were also computed. They reported good agreement between the values obtained from experiments and those predicted by the theory.

Tauchert and Guzelsu [18] have used ultrasonic pulse technique to study dispersive relations of plane harmonic wave in boron epoxy composites. Acoustical velocities in longitudinal and transverse directions are measured for different frequency pulses. The relationship between the group velocities and frequency revealed the dispersion characteristics of the material..The static modulii for the composites were also computed.

Munson and Schuler [19] proposed a mechanical theory to study the wave propagation in laminated media. Both longitudinal and transverse propagation are considered. linear elastic models and nonlinear hydrodynamic models are proposed for different materials and pressure levels. They have found out that propagation normal to the laminates is identical to propagation along the laminates.

Hemann and Baaklini [20] have used ultrasonic technique to show that stress wave propagation is dependent on stress levels.Attenuation of stress waves was found to be related to tensile stress levels in the composite. The stress levels were kept within the elastic limit and no macro level damage was observed.Wave speeds in the graphite epoxy material were found to

vary between 2700 m/s and 6800 m/s for pulses of frequencies from 15 kHz to 1.9 MHz. It was observed that wave speeds increase slightly at all tested frequencies with increase in stress levels.

Recent developments in digital signal processing and electronic hardware engineering has facilitated better extraction and analysis of experimental data and thus has enabled application of the same for defect identification and quantification.

Teti and Caprino [21] have used digitized ultrasonic data for analysing defects like porosity delamination etc.,in thick glass fiber composites.Complete waveform digitization and analysis has enabled them to localize and to identify the defects.

Dayal and Kinra [22] developed a new technique for ultrasonic evaluation of thin specimens. Use has been made of FFT methods with conventional ultrasonics for measuring phase velocity, group velocity and attenuation of thin (sub millimeter and sub wavelength thick) specimens. The technique has been found to be suitable for dispersive materials also. Effect of number of transverse cracks on the attenuation has been studied in graphite epoxy laminates. Attenuation seems to be increasing monotonically and significantly with increasing number of cracks. Also phase velocity seemed to remain constant with increase in damage.

Ultrasonic technique has been used to study the fatigue behavior of composites [24,25]. Variation in time and frequency domain parameters of the output pulse have been studied to associate them to specific changes in the structure and characteristics of the material. So far one to one correspondence between the two has not been established conclusively.

From the literature survey it is evident that much of

analytical work has been concentrated on studying dispersion relations and modal frequencies and displacements in composites. Main difficulties that arise while analysing these materials are their dispersive nature, heterogeneities at the micro level, viscoelastic effects of matrix etc. Some Work has been done to overcome these difficulties using alternate models such as control volume analysis [26] etc. Application of effective stiffness and effective modulus theories to study dynamic behavior of composites are reported in references 27 and 28. Both discrete and continuum models are considered. Tewary [27] has extended the same to study the effects of defective fibers on wave propagation.

1.6. PRESENT WORK :

In the present work attempt has been to study the harmonic wave propagation in composite materials. Emphasis is on determination of dispersion relation in the plane of fibers and in the thickness direction. Analysis is done by two methods.

(a) By modelling the composite material in a such manner that fibers are uniformly spaced

(b) By modelling the composite medium as a uniform homogeneous medium with appropirate average properties.

Concepts of Floquet's theory and finite element method are used in the analysis. Dispersion relations are determined for various material properties.

CHAPTER 2

BASICS OF WAVE PROPAGATION AND ITS APPLICATION IN COMPOSITE MATERIALS

In this chapter a brief overview of the underlying principles of wave propagation and basic analysis procedure of composite material is given .

2.1.1.WAVE PROPAGATION :

Wave or disturbance is a process or an influence like some form of energy which moves through the material with a finite velocity, without causing any bulk transfer of the material as a whole.

Waves may be either standing or travelling waves. In vibration problems one encounters standing waves, such as in the case of vibrations of strings. In case of a shock or impact loading the travelling waves are generated. Depending on the form of the solution desired for the wave equation one gets a standing wave or a travelling wave solution. Separation of variables technique yields a standing wave solution whereas the method of characteristics gives a progressive wave solution.

The governing equation of wave motion is of the form

$$\frac{\partial^2 \psi}{\partial t^2} = C^2 \frac{\partial^2 \psi}{\partial x_i \partial x_j} , \quad \psi = \psi(x_i, t) \tag{2.1}$$

where Ψ is some disturbance, c, is a physical constant and x_i the coordinates, x_1 x_2 x_3 in x,y,z directions. Choosing $\psi(x,t) = X(x) T(t)$ leads to a standing wave solution and $\psi = \psi(x-ct)$, gives a travelling wave solution.

2.1.2 HARMONIC WAVES :

If the solution is of the form:

$$\psi = A \sin \left[\frac{2\pi}{\lambda} (x \pm ct) \right] \tag{2.2}$$

where λ is the wave length, c the velocity of propagation and $\frac{2\pi}{\lambda}$ is the wave number giving the number of waves per cycle , a harmonic wave solution is obtained. The relation between wave number **k** , circular frequency ω and velocity of propagation c is

$$\omega = kc \tag{2.3}$$

2.1.3 PHASE AND GROUP VELOCITIES :

The velocity with which the points of same phase move in a medium is called as phase velocity. This is same as c in equation 2.3 .

The velocity with which the modulations or beats propagate in the material is termed as group velocity. It is denoted by c_g and is given by the relation

$$c_g = \frac{d\omega}{dk}, \tag{2.4}$$

Modes are patterns of motion having a property that at any point in it, the body moves perfectly sinusoidally and all points move with same frequency.

Dispersion is the frequency dependent phenomenon of a medium wherein the wave front undergoes a change in shape as it propagates in the material. Flexural waves and waves in anisotropic material are certain examples of dispersive waves.

2.1.4 TYPES OF WAVES :

There are different types of waves called longitudinal waves, transverse waves ,plate waves, surface waves etc. In longitudinal waves the direction of particle motion is same as the propagation direction. If the direction of particle motion is perpendicular to the direction of propagation then the wave is called as shear or transverse wave. Sound waves travelling in air are examples of longitudinal waves and electromagnetic waves are examples of transverse waves.

Waves are also classified as bulk waves ,surface waves etc.. Longitudinal and shear waves travelling in an infinite medium are examples of bulk waves. Here the surface effects are not felt by the waves.

Surface waves are waves propagating near and on the surface of the medium. A plane surface wave is called a Rayliegh wave in which the particle motion will be elliptical. Waves caused by the movement of earth's crust and travelling on the surface are examples of Rayleigh waves. Surface waves travelling in thin plates and shells are termed Lamb waves or plate waves.

2.1.5 FLOQUET THEORY :

Floquet theory concerns with the solutions of differential equations with periodic coefficients. Consider a differential

equation of the form ,

$$a_0(x)\, y''(x) + a_1(x)\, y'(x) + a_2(x)\, y(x) = 0 \qquad (2.5)$$

in which the coefficients $a_r(x)$ are complex valued, piecewise continuous and periodic, all with the same period 'd':
i.e., $a_r(x+d) = a_r(x)$, ($0 < r < 2$) and 'd' is a non zero real constant. It can be observed that if $\psi(x)$ is a non trivial solution of equation 2.5 then $\psi(x+d)$ is also a solution. Floquet theory.provides a proof for the property that

$$\psi(x+d) = \rho\, \psi(x) \qquad (2.6)$$

where ρ is a non zero constant.

This result finds application in solution of the wave equation and it simplifies the solution appreciably, because it facilitates application of concept of unit cells and periodic boundary conditions to the problem.

2.1.6. UNIT CELL APPROACH :

As mentioned above Floquet theory simplifies the analysis of the wave problem in a periodic medium to the analysis of a small a portion of the spatial domain, called a unit cell which satisfies the following conditions.

1. The medium under consideration can be discretised into identical cells representing one period of the medium, 'd'.
2 All the characteristics of the medium should be represented by the cell.

Fig.2.1 represents a one dimensional medium discretised into unit cells. If it is assumed to contain N number of particles in each cell, the particle N+p is said to be related to particle N+q

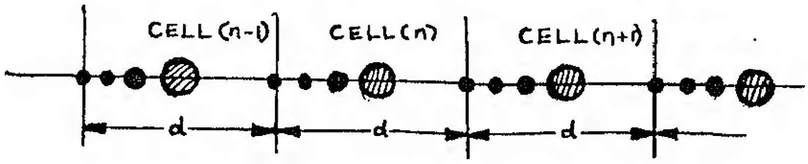


Fig. 2.1 Unit cell in a periodic one dimensional medium

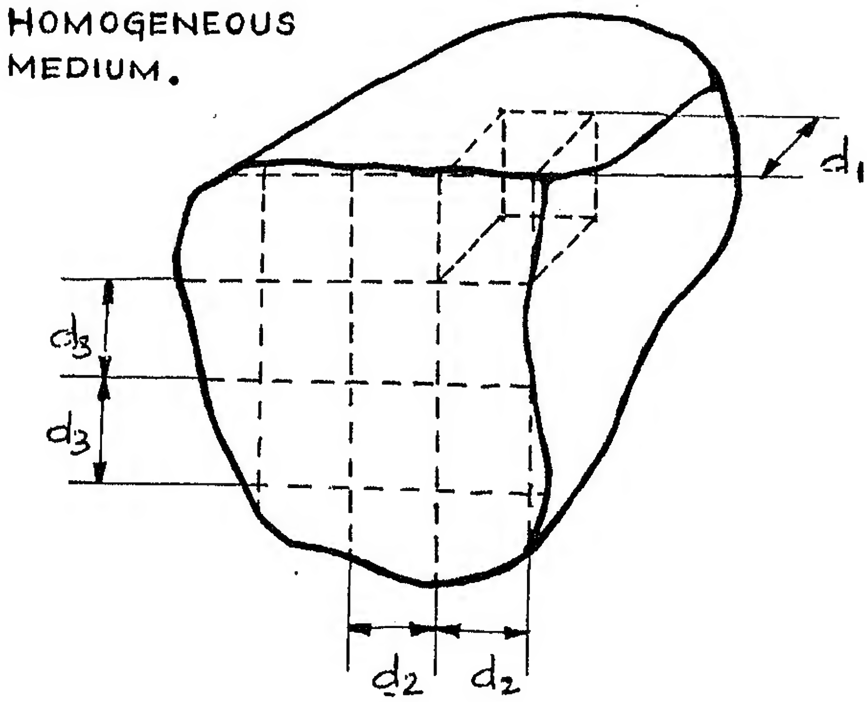


Fig. 2.2 Unit cell an isotropic
3 - D infinite medium

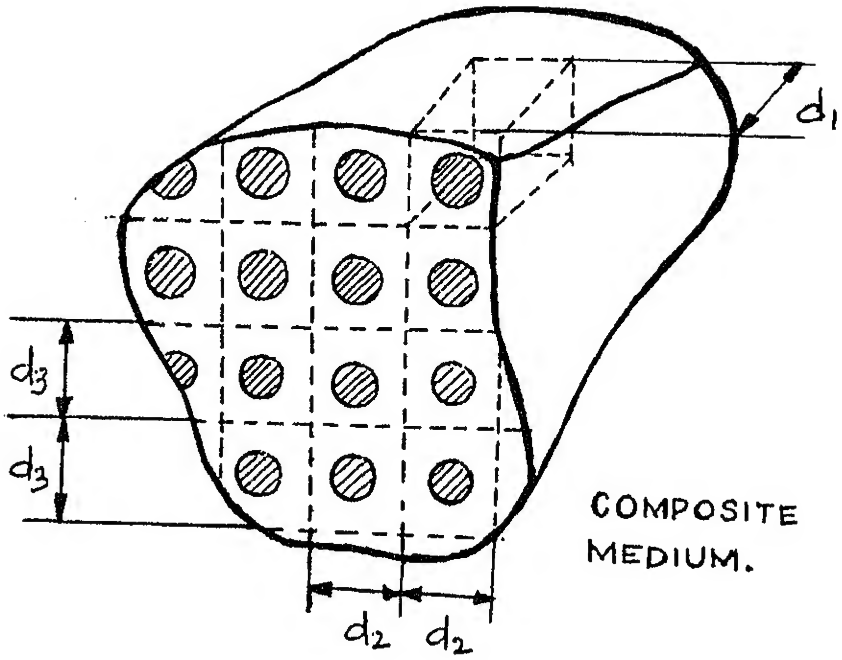


Fig. 2.3 Unit cell in an infinite
3-D infinite medium

in the same way as particle p is related to particle q. It can be seen that if N approaches infinity then the discrete medium becomes continuous.This concept can be extended to two and three dimensions also. For a general three dimensional medium the representation of unit cell is shown in the figure 2.2.

In a homogeneous material there is no physical significance for an unit cell. For homogeneous crystalline materials the lattice can be likened to an unit cell. In amorphous materials it is difficult to identify a basic representative and repetitive unit.

A uniform composite material can be idealized as the fibers being arranged in regular array and embedded in a matrix material. Here the unit cell can be seen to be a region shown as dotted rectangle (Fig.2.3). In some cases the unit cell may be hexagonal. It is evident from the figure that the cell possesses all the engineering properties of the medium . In the present study rectangular type of unit cells are assumed to discretise the medium.

2.2. WAVE CHARACTERISTICS IN COMPOSITE MATERIALS :

Wave characteristics in homogeneous materials have been studied extensively [29,30,31]. But only limited understanding of the same is available in the case of composite materials. Material non homogeneity, layered structure ,anisotropy etc., that differentiate the composites from isotropic materials are reflected in their wave propagation characteristics. Figures 2.4,2.5 [16] show the difference in wave propagation characteristics of the two types of materials.

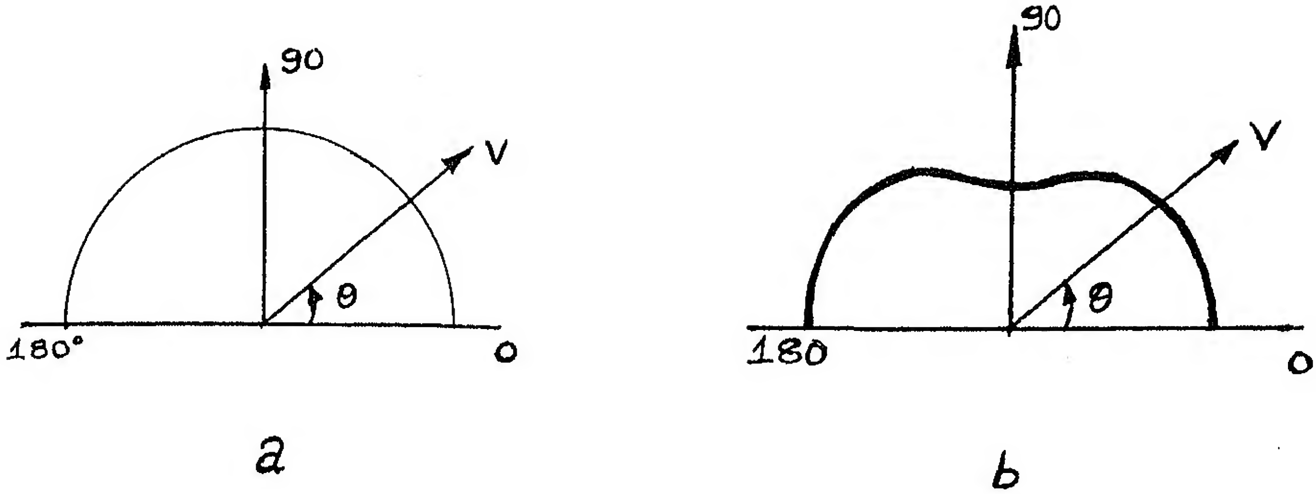


Fig. 2.4 Spherical wave velocity profile for a homogeneous isotropic material.

Fig. 2.4b A typical wave velocity profile for a composite material.

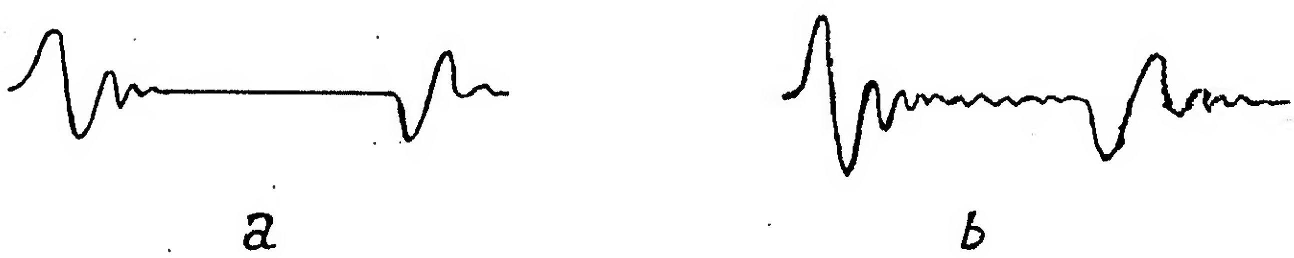


Fig. 2.5a Typical A-mode display for a homogeneous isotropic material.

Fig. 2.5b Typical A-mode display for a composite material.

Composites materials are dispersive in nature due to their inherent anisotropy and non homogeneity. Thus the wave profile is constantly changing and it is difficult to separate different frequency components of the wave. This also implies that it is difficult to define phase velocity for these materials. The velocity that can be measured is the group velocity.

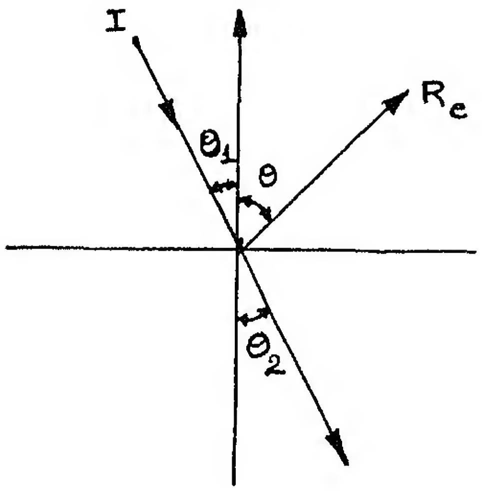
The acoustic impedance represented by ρc is difficult to obtain in the case of composite materials as c is a function of frequency. Since this property is important in assessing the propagating characteristics it is often necessary to define an average value of the acoustic impedence for specific cases in order to study the characteristics.

When a wave passes through an interface of two media some portion of it get reflected and some get transmitted (refraction). The refraction formula is also known as Snell's law, written mathematically as

$$c_2 \sin\theta_1 = c_1 \sin\theta_2 \qquad (2.7)$$

where θ_1 and θ_2 are shown in Fig. 2.6. This cannot be easily applied to composite materials as velocity is dependent on the direction and the associated mode conversion .

Attenuation levels are much higher in composites than in isotropic materials. This effect is more pronounced at higher frequencies as the associated wavelength approaches the size of the fiber . Local heterogeneity is also a factor that causes attenuation due to scattering of waves etc..



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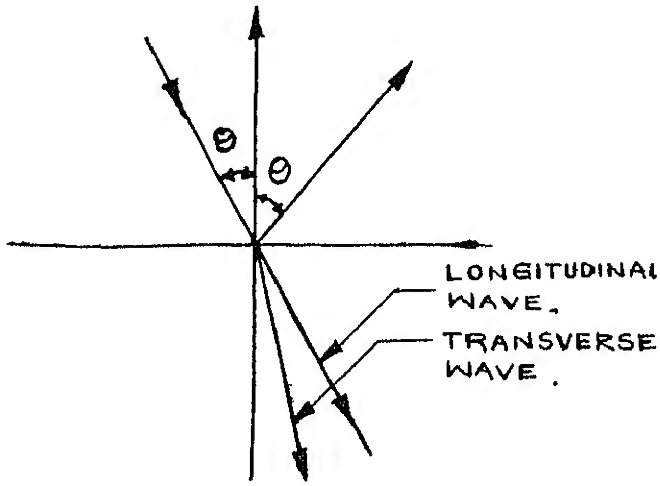


b

Fig. 2.6a. Refraction of waves at an interface between two isotropic media.

Fig. 2.6b. Refraction and mode conversion of waves at an interface of a composite medium

CHAPTER 3

PROBLEM DEFINITION AND FINITE ELEMENT ANALYSIS

In this chapter the present problem is defined and necessary assumptions made are described. Finite element formulation of the problem and its analysis procedure are highlighted.

3.1. PROBLEM DEFINITION :

In the present work attempt has been made to study wave propagation in composite materials. Attention has been focused on two dimensional harmonic wave propagation in fiber reinforced composite materials.

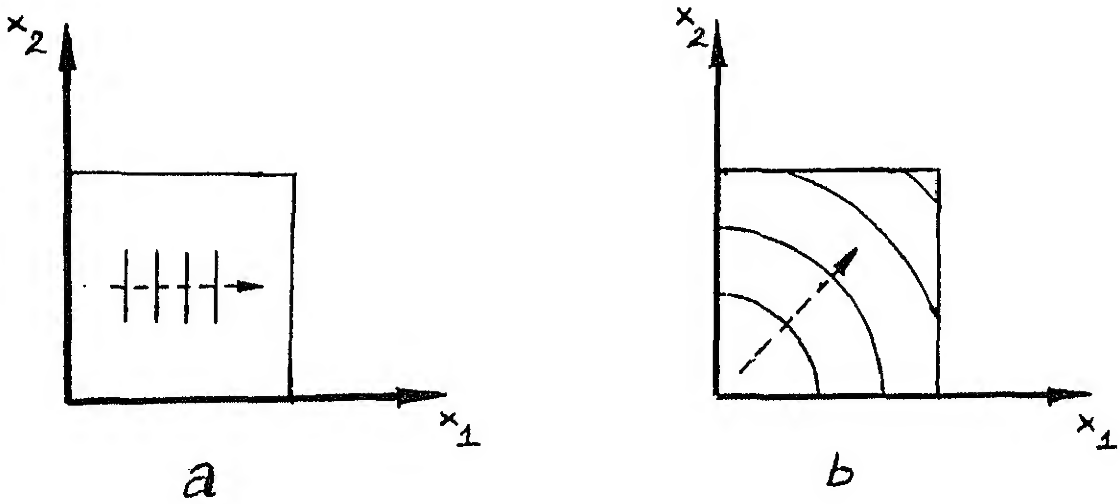
In two dimensional wave propagation two cases are of interest viz., the propagation in the longitudinal plane containing fibers and in the thickness plane as shown in figure 3.1. The waves propagating in these two planes can be either parallel or circular/elliptical. Hence the cases considered are

a) Wave propagation in plane of the plate (Fig. 3.1) with the particle motion confined to this plane alone.

b) Wave propagation in thickness direction (Fig. 3.2) with the particle motion confined to this plane alone.

Following simplifying assumptions are made in the analysis:

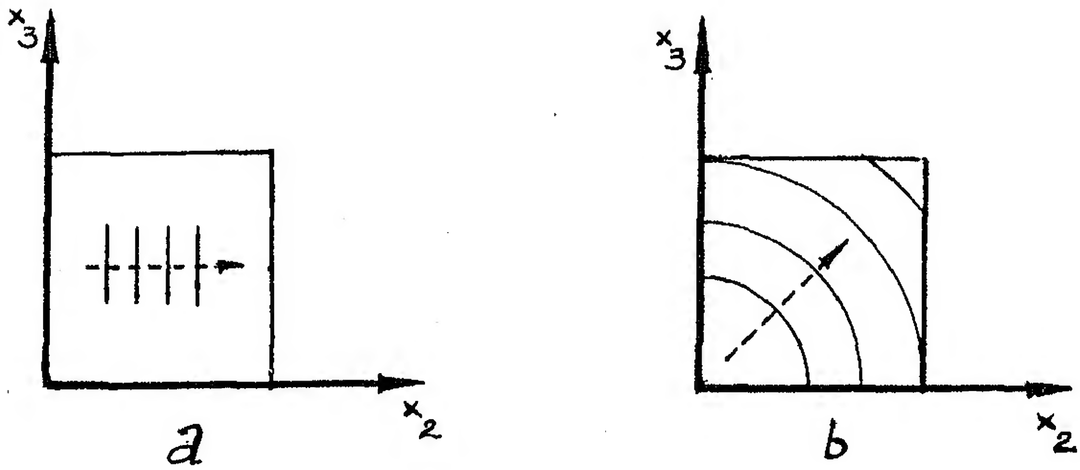
1. The composite material is defect free and has fibers distributed uniformly.
2. The continuum can be represented by unit cells as explained in section 2.1.6
3. The material behavior is to be elastic in nature.



FI Fig. 3.1 Two dimensional wave propagation in $x_1 - x_2$ plane.

3.1a Plane waves

3.1b Circular waves



FIG, Fig. 3.2 Two dimensional wave propagation in $x_2 - x_3$ plane

3.2a Plane waves

3.2b Circular waves

4. The medium is of infinite extent in order to avoid boundary effects such as reflection and refraction.
5. Case (a) is treated as plane stress case and case (b) as plane strain case.

The periodic nature of medium is illustrated in figure 2.3 and typical unit cell. Because of the periodic structure of the medium Floquet's theory can be applied giving a simple and feasible method of analysis.

The governing equation of motion is of the form

$$\frac{1}{2}\frac{\partial}{\partial x_j}\left[C_{ijkl}\left(\frac{\partial u_k}{\partial x_l}+\frac{\partial u_l}{\partial x_k}\right)\right]=\rho\,\frac{\partial^2 u_i}{\partial t^2} \qquad (3.1)$$

where ρ is mass density , C_{ijkl} is elastic stiffness coefficient $(i,j,k,l=1,2,3)$ and u_i are generalized displacements (u,v,w components in x,y,z directions respectively).The displacements are assumed to satisfy the harmonic relation ,

$$u(x,y,t) = u(x.y)e^{-i\omega t} \qquad (3.2).$$

Which results in an eigen value problem. This is solved for natural frequencies,ω and associated mode shapes (eigen vectors) as a function of wave number ζ.

3.2. FINITE ELEMENT FORMULATION :

The wave equation 3.1 is solved by FEM using the Galerkin method [1]. Assuming an approximate solution for u_k in equation 3.1 as $\hat{u}_k$ and applying weighted residual principle, the residual can be written as

$$R(\hat{u}_k, w) = \int_D w_i \left\{ \frac{1}{2} \frac{\partial}{\partial x_j} \left[C_{ijkl} \left(\frac{\partial \hat{u}_k}{\partial x_l} + \frac{\partial \hat{u}_l}{\partial x_k} \right) \right] - \rho \frac{\partial^2 \hat{u}_i}{\partial t^2} \right\} dA \quad (3.3)$$

where w =w(x,y) are the weighting functions and R ,the residual over the domain D. Considering the first term in R and applying divergence theorem ,it takes the form:

$$\frac{1}{2}\int_L w_i \left[C_{ijkl} \left(\frac{\partial \hat{u}_k}{\partial x_l} + \frac{\partial \hat{u}_l}{\partial x_k} \right) \right] \hat{n}_j \, ds - \int_D \frac{\partial w_i}{\partial x} \left[C_{ijkl} \left(\frac{\partial \hat{u}_k}{\partial x_l} + \frac{\partial \hat{u}_l}{\partial x_k} \right) \right] dA$$

(3.4)

The first term in expression 3.4 is the boundary term and as w_i along the boundary are assumed to be zero, the residual R can be written as

$$R = -\int_D \left\{ \frac{\partial w_i}{\partial x} \left[C_{ijkl} \left(\frac{\partial \hat{u}_k}{\partial x_l} + \frac{\partial \hat{u}_l}{\partial x_k} \right) \right] - \rho \frac{\partial^2 \hat{u}_i}{\partial t^2} \right\} dA \quad (3.5)$$

Using equation 3.2 in the residual , R Eqn.3.5 takes the form :

$$R = \int_D \left\{ \frac{\partial w_i}{\partial x} \frac{1}{2} \left[C_{ijkl} \left(\frac{\partial \hat{u}_k}{\partial x_l} + \frac{\partial \hat{u}_l}{\partial x_k} \right) \right] + \rho w_i \Omega^2 \hat{u}_i \right\} dA \quad (3.6)$$

3.3. FINITE ELEMENT DISCRETIZATION :

Figure 2.3 shows the domain and a typical unit cell, the origin of which is located at $n_1 d_1 + n_2 d_2$ from the system or domain origin. An arbitrary cell is discretized into rectangular 8-noded isoparametric elements (Fig. 3.3) . The generalized displacements at every node are u and v in x and y directions respectively, satisfying C^0 continuity.

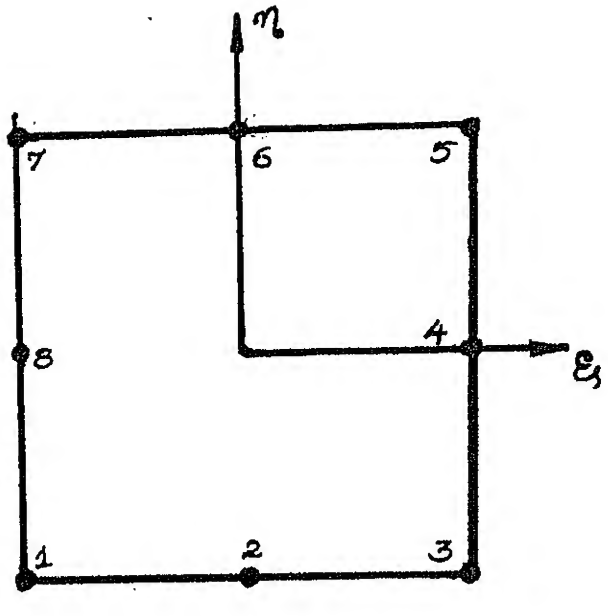


Fig. 3.3 An 8 - noded isoparameteric element.

We have :

$$w = \Sigma\, w_i\, N_i$$

$$u = \Sigma\, u_i\, N_i$$

$$v = \Sigma\, v_i\, N_i \qquad (3.7)$$

where N_i are the nodal shape functions ,in natural coordinates ξ and η (figure 3.3), and i the node number.

$N_i^e = \frac{1}{4}(1 + \xi\, \xi_i)(1 + \eta\, \eta_i)(-1 + \xi\, \xi_i + \eta\eta_i)$

$i = 1,3,5,7$

$N_i^e = \frac{1}{2}(1 - \xi^2)(1 + \eta\, \eta_i)$

$i = 2,6$

$N_i^e = \frac{1}{2}(1 - \eta^2)(1 + \xi\, \xi_i)$

$i = 4,8 \qquad (3.8)$

The nodal displacements u_j, v_j are , for j^{th} node:

$$u_j = U_j\, e^{i(n_1 k + n_2 k_2 - \omega t)}$$

$$v_j = V_j\, e^{i(n_1 k + n_2 k_2 - \omega t)} \qquad (3.9)$$

In equation 3.9 $n_1 k_1 + n_2 k_2$ represent origin of the unit cell in terms of wave vector. If U_j and V_j are expressed as follows,

$$U_j = U1_j + i\ U2_j \quad ;$$

$$V_j = V1_j + i\ V2_j \qquad (3.10)$$

the nodal degree of freedom vector takes the form:

$$\{\ \psi\ \}_j = \lfloor\ U1,\quad U2,\quad V1,\quad V2\ \rfloor_j \qquad (3.11)$$

3.4 ELEMENTAL MATRICES :

The nodal stiffness and mass matrices are of the form [32]:

$$[\,K\,]_{ij} = t \int_D [B]_i^T \; [D]_{ij} \; [B]_j \; dA$$

$$[\,m\,]_{ij} = \int_D \rho \; [N]_i^T \; [N]_j \; dA \tag{3.12}$$

where $[B]_i$ is the strain matrix and [D] is the coefficient matrix of the form:

$$[B]_{ij} = \begin{bmatrix} N_{i,x} & N_{i,x} & \emptyset & \emptyset \\ \emptyset & \emptyset & N_{i,y} & N_{i,y} \\ N_{i,y} & N_{i,y} & N_{j,x} & N_{j,x} \end{bmatrix}$$

$$[D]_{ij} = \begin{bmatrix} D_1 & D_2 & D_3 \\ & D_5 & D_6 \\ \text{Sym} & & D_9 \end{bmatrix} \tag{3.13}$$

From equations 3.12 and 3.13 nodal stiffness and mass matrices are 4X4 and elemental mass and stiffness matrices are 32X32 in size.

The integrals in equation 3.12 are evaluated using numerical integration . Using Gauss quadrature integration rule they can be expressed as :

$$[K]_{ij} = \sum_p \sum_q W_p W_q [B]_i [D]_i [B]_j |J|$$

$$[M]_{ij} = \sum_p \sum_q W_p W_q \rho \; N_i N_j |J| \tag{3.14}$$

where W_p and W_q are the weights of the quadrature, p and q are the number of points in ξ and η directions and $|J|$ is the jacobian of

transformation written as:

$$|J| = (x_{,\xi}\ y_{,\eta} - x_{,\eta}\ y_{,\xi}) \qquad (3.15)$$

It can be seen from Eqn.3.14, 2x2 integration is necessary for stiffness matrix calculation and 3x3 integration for mass terms. The summation is to be performed for 4 times for every stiffness term and nine times for every mass term. The special nature of [D] can be made use of in order to simplify computation. Using an efficient algorithm [32] one can form an intermediate matrix containing the summed up products of shape function derivatives, of the form :

$$[A]_{ij} = \begin{bmatrix} \underset{(A_1)}{\Sigma\Sigma N_{i,x} N_{j,x}} & \underset{(A_2)}{\Sigma\Sigma N_{i,x} N_{j,y}} \\ & \\ \underset{(A_3)}{\Sigma\Sigma N_{i,y} N_{j,x}} & \underset{(A_4)}{\Sigma\Sigma N_{i,y} N_{j,y}} \end{bmatrix}_{ij} W_p\ W_q\ |J|. \qquad (3.16)$$

It is enough if we compute $[A]_{ij}$ by summation at four gauss points and then compute $[K]_{ij}$,thus saving appreciable amount of computation time. It is possible to simplify further the computation of $[K]_{ij}$ using the special nature of $[B]_i$. By forming another matrix $[S]_{ij}$ written as follows,

$$[S]_{ij} = \begin{bmatrix} \underset{(S11)}{D_1A_1+D_3(A_2+A_3)+D_9A_4} & \underset{(S12)}{D_2A_2+D_3A_1+D_6A_4+D_9A_3} \\ \underset{(S21)}{D_2A_3+D_3A_1+D_6A_4+D_9A_2} & \underset{(S22)}{D_5A_4+D_6(A_2+A_3)+D_9A_1} \end{bmatrix}_{ij} \qquad (3.17)$$

$[K]_{ij}$ can be written as follows:

$$[\,K\,]_{ij} = \begin{bmatrix} S_{11} & S_{11} & S_{12} & S_{12} \\ S_{11} & S_{11} & S_{12} & S_{12} \\ S_{21} & S_{21} & S_{22} & S_{22} \\ S_{21} & S_{21} & S_{22} & S_{22} \end{bmatrix}_{ij} \qquad (3.18)$$

Similarly for $[M]_{ij}$ given by Eqn. 3.14, one can form an intermediate matrix that stores the summed up product of shape functions, N_i and N_j and then express $[M]_{ij}$ as follows.

$$[\,M\,]_{ij} = \begin{bmatrix} L_{ij} & \emptyset & \emptyset & \emptyset \\ \emptyset & L_{ij} & \emptyset & \emptyset \\ \emptyset & \emptyset & L_{ij} & \emptyset \\ \emptyset & \emptyset & \emptyset & L_{ij} \end{bmatrix} .\rho$$

where $$L_{ij} = \sum_p \sum_q N_i \; N_j \; |J| \; W_p \; W_q \qquad (3.19)$$

Thus to compute elemental stiffness and mass matrices $[K]^e$ and $[M]^e$ it is only required to choose proper i and j and compute Eqn.3.18 and 3.19. Moreover if all the elements have same shape and size it is enough if we compute them once and use it for all the elements. This reduces the computation involved by an order of magnitude.

The equation (3.1) of motion can be rewritten as

$$\left[\left[K^{C} \right] - \omega^{2} \left[M^{C} \right] \right] \left\{ \psi \right\} = \emptyset$$

The elemental matrices are assembled to get the global matrices. The boundary conditions are applied to global matrices to yield a real eigen value problem. This is solved for various values of ζ to get eigen values (frequencies) and mode shapes.

3.5 BOUNDARY CONDITIONS :

From Floquet's theory the boundary condition for the considered unit cell can be written in general as (figure 2.3) follows. If L,R,B,T subscripts denote left, right, bottom and top boundaries of the unit cell respectively, the generalized displacements { ψ } along the boundary have the following relations.

$$\begin{aligned} \{ \psi \}_R &= e^{i(k_i d_i)} \{ \psi \}_L \\ \{ \psi \}_T &= e^{i(k_i d_i)} \{ \psi \}_B \end{aligned} \qquad (3.21)$$

Where k_1, k_2 are the wave vectors in x_1, x_2 directions respectively.

It may be noted that that for free wave propagation in an infinite medium the nodal forces are zero. Thus for the equilibrium of unit cell the nodal forces are given by

$$\{ F_R \} = - e^{i(k_i d_i)} \{ F_L \} \quad ; \quad \{ F_T \} = - e^{i(k_i d_i)} \{ F_B \} \qquad (3.22)$$

Thus from Eq. (3.21) it can be seen that U_R, V_R, U_T, V_T can be expressed in terms of U_L, V_L, U_T, V_T and can be eliminated from the set of unknowns as follows.

$$\left\{ \left[K^{C} \right] - \omega^{2} \left[M^{C} \right] \right\} \left\{ \psi \right\} = \left\{ F \right\}$$

where

$$\left\{ \psi \right\}^{T} = \left\{ \left\{ \psi \right\}_{L} \; \left\{ \psi \right\}_{B} \; \left\{ \psi \right\}_{I} \; \left\{ \psi \right\}_{R} e^{ik_1 d_1} \left\{ \psi \right\}_{T} e^{ik_2 d_2} \right\}$$

$$\left\{ F \right\} = \left\{ \{F\}_{L} \quad \{F\}_{L} \quad \{\emptyset\}_{B} \quad -\{F\}_{L}\, e^{ik_1 d_1} \quad -\{F\}_{B}\, e^{ik_2 d_2} \right\}^{T} \tag{3.23}$$

The stiffness and mass matrices can be partitioned as follows:

$$[K]^{c} = \begin{bmatrix} K_{LL} & K_{LB} & K_{LI} & K_{LR} & K_{LT} \\ & K_{BB} & K_{BI} & K_{BR} & K_{BT} \\ & & K_{II} & K_{IR} & K_{IT} \\ \text{Sym} & & & & \\ & & & & K_{TT} \end{bmatrix} \tag{3.24}$$

$$[M]^{c} = \begin{bmatrix} M_{LL} & M_{LB} & M_{LI} & M_{LR} & M_{LT} \\ & M_{BB} & M_{BI} & M_{BR} & M_{BT} \\ & & M_{II} & M_{IR} & M_{IT} \\ & \text{Sym} & & & M_{TT} \end{bmatrix} \tag{3.25}$$

Further simplification is done using the relations Eq.3.24 and 3.25 . The procedure is illustrated by considering the right and left boundary relationships.

The right hand boundary degrees of freedom when expressed as a function of left hand boundary degree of freedom the system of equations simplify as

$$\begin{bmatrix} K_{LL} + K_{RR} + e^{\mu} K_{LR} + e^{-\mu} K_{RL} & K_{LI} + e^{\mu} K_{R} \\ K_{IL} + e^{\mu} K_{IR} & K_{II} \end{bmatrix} \begin{Bmatrix} \psi_L^{c} \\ \psi_I^{c} \end{Bmatrix} - \omega^2 \begin{bmatrix} M_{LL} + M_{RR} + e^{\mu} M_{LR} + e^{-\mu} M_{RL} & M_{LI} + e^{\mu} M_{RI} \\ M_{IL} + e^{\mu} M_{IR} & M_{II} \end{bmatrix} \begin{Bmatrix} \psi_L^{c} \\ \psi_I^{c} \end{Bmatrix} = \emptyset \qquad (3.26)$$

where $\mu = ik\ d$

Similarly the relationship between top and bottom nodal degrees of freedom can be used resulting in an expression similar to equation 3.26. Then Eqn. 3.2Ø can be written as,

$$\left[\left[K\ (\mu_1,\mu_2)^{\ c} \right] - \omega^2 \left[M\ (\mu_1,\mu_2)^{\ c} \right] \right] \left\{ \psi \right\}_T = \left\{ \emptyset \right\} \quad (3.27)$$

where:

$$\left\{ \psi \right\}_T = \left\lfloor \{\psi_L\} \quad \{\psi\}_B \quad \{\psi_I\} \right\rfloor \qquad \mu_1 = ik_1 d_1 \ , \quad \mu_2 = ik_2 d_2$$

and K and M matrices are real function of complex variable le μ_1 μ_2.

Thus Equation 3.27 represents a complex eigen value problem, the complex quantities coming from the boundary conditions.. It can also seen that the matrices K and M appear as complex conjugates and hence are Hermitian.

As it is difficult to solve large eigen system with complex elements, it is required to convert them into system of real matrices. The simplification procedure is given by Wilkinson [34] and by J.V Sastry [35] .

Let

$$\left[K\ (\mu_1, \mu_2) \right] = \left[K^r + i\ K^i \right]; \ \left[M\ (\mu_1,\mu_2) \right] = \left[M^r + i\ M^i \right]$$

$$\{ \psi \} = \{ \psi^r + i\ \psi^i \} \quad (3.28)$$

where r, i denote real and imaginary parts, Substituting Eqn.3.28 in Eqn.3.27 the Eqn. 3.27 takes the form

$$\left[\left[K^r + i\ K^i \right] - \omega^2 \left[M^r + i\ M^i \right] \right] \left(\psi^r + i\psi^i \right) = \emptyset \quad (3,29)$$

Separating the real and imaginary parts and combining the two sets

of equations, Eq. 3.24 can be expressed as

$$\left\{ \begin{bmatrix} K^r & -iK^i \\ K^i & K^r \end{bmatrix} - \omega^2 \begin{bmatrix} M^r & -iM^i \\ M^i & M^r \end{bmatrix} \right\} \begin{Bmatrix} \psi^r \\ \psi^i \end{Bmatrix} = \emptyset \qquad (3.30)$$

Since the matrices [K] and [M] are Hermitian ,

$[K^i] = -[K^i]^T$, $[M^i] = -[M^i]^T$

Thus equation 3.29 represents a real symmetric eigen value problem, of rank 4N where 'N ' is the number of independent nodes belonging to the unit cell. The equation 3.30 can be solved for natural frequencies 'ω 'and eigen vectors { ψ } by specifying the values of k_1, K_2 and d_1, d_2.

3.6 COMPUTER IMPLEMENTATION :

The global stiffness and mass matrices are obtained by assembling the elemental matrices. These elemental matrices are in turn obtained by calculating stiffness and mass sub- matrices for any pair of nodes in an element and then grouping them.

Assembling and storage of these large matrices are effected using profile storage or skyline system [36]. In order to apply the boundary conditions the boundary nodes are given the same node number for those of which the degrees of freedom are related. A special connectivity is given for the assembly purpose. Another connectivity and node numbering system is also provided to calculate the shape functions, mesh generation and calculation of the jacobian etc..

Subspace iteration technique detailed in Ref36 is used for obtaining lowest few eigen values and eigen vectors. Because of the free-free boundary conditions for the cell, zero eigen values appear corresponding to the possible rigid body modes. Hence a shift is applied for the eigen values [1] in order to over come the zero eigen values.

CHAPTER 4

RESULTS AND DISCUSSION

In this chapter the examples for analysis and the results obtained are presented. A discussion on the results obtained is also presented. The following cases are solved to obtain dispersion relations(frequency spectrum) and mode shapes as a function of wave number ζ.

4.1 EXAMPLES :

Two types of materials viz, a homogeneous isotropic material and composite material are investigated. Wave propagation both in the plane of the plate and in the thickness plane (plane stress and plane strain case) is studied. An isotropic material is considered in order to validate the analysis procedure. Two types of fiber reinforced composite materials are considered for investigating the free wave propagation in composite materials. The materials of fibers and matrix are so chosen that the ratios of fibre and matrix stiffness are different. Properties of materials considered here are presented in Table 4.1.

4.2. FINITE ELEMENT MESH :

The unit cell size is chosen to be $d_1=d_2=$ 1 mm and is discretized using 8-noded isoparametric elements. The cell is divided into 16 elements, 4 in each direction (Fig 4.1.). The total number of nodes in the unit cell including those on the

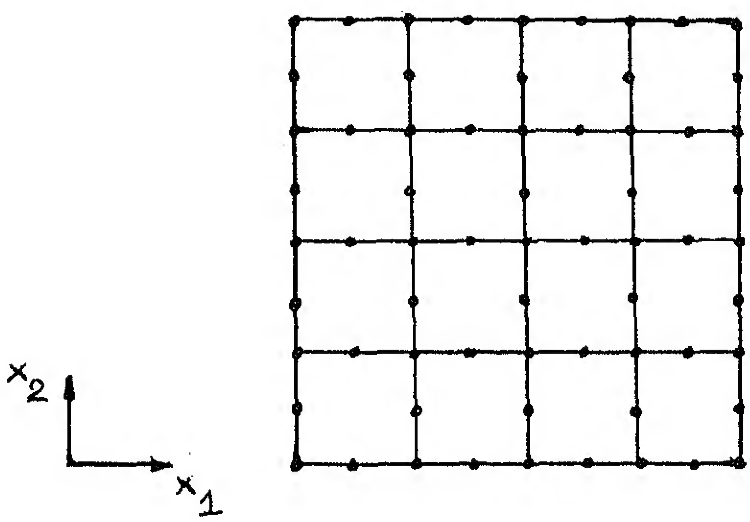


Fig. 4.1 Finite element mesh for the unit cell in an infinite medium.

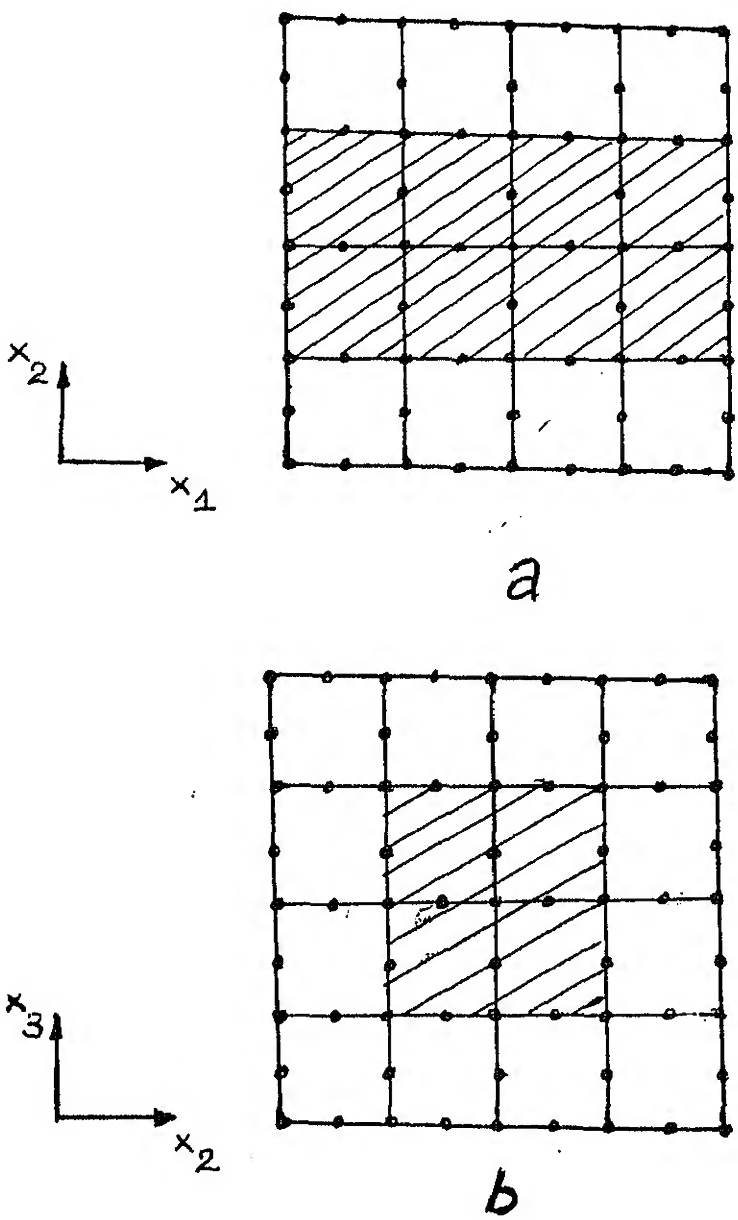


FIG Fig. 4.2 Finite element mesh for the unit cell in composite medium

4.2a For propagation in $x_1 - x_2$ plane

4.2b For propagation in $x_2 - x_3$ plane

right and top boundaries is 96. But if the nodes on the right and top boundaries are considered as those belonging to the adjacent cell but interacting with the considered cell, the number of nodes then are 75, belonging to the cell.

In composite materials, the unit cells for propagation in the plane of fibers (plane stress case) and in the thickness plane (plane strain case) are different and are illustrated in Figs. 4.2 a,b.. From the Fig. 4.2 a,b the volume fraction of fibres and matrix are as follows:

(a) Plane stress case ; $V_f = 0.5$

(b) Plane strain case ; $V_f = 0.3$

4.3 CONVERGENCE CRITERION :

In the subspace iteration the following convergence criterion for eigen values is used :

$$\left| \frac{\lambda_i^{k+1} - \lambda_i^{k}}{\lambda^{k}} \right| \leq \varepsilon \qquad (4.1)$$

where λ_i^k denotes the i^{th} eigen value after the k^{th} iteration and ε is the tolerance limit specified.

In the present analysis two values of tolerance were tried. The tolerance limit of 10^{-3} and 10^{-6} were provided for the eigen values.

4.4 ISOTROPIC MATERIAL

An isotropic material with E= 30 GPa and ν =0.3 and ρ = 700 kg/m^3,.Sixteen elements were used to discretize the unit cell. For this case ten lowest eigen values and corresponding eigen vectors are obtained for various values of wave number ranging from (0.001-2.00). These are obtained for both plane stress and plane strain cases. The results for plane stress case are presented in Table 4.2. and Fig 4.3. The eigen frequencies are normalised with respect to the third lowest frequency at ζ' = 0.001. The normalised frequencies for plane stress case are given in Table 4.2 and the frequency spectrum is plotted in Fig 4.3.

From figure 4.3 and table 4.2 following points may be noted.

1) The frequency spectrum is symmetric about ζ = 1.0 . This may be attributed to the fact that at ζ = 1 the half wavelength is equal to the unit cell dimension d.

2) There are two types of branches in the frequency spectrum viz.,the acoustical branch and the optical branch [29].

In the Brillouin sense they are defined based on the way individual particles move in the direction of wave propagation. If all the points (Fig.4.4) within half wavelength in the direction of propagation are having same orientation with respect to direction of propagation the corresponding branches are called as acoustical branches. If the particle orientation is different at every point then it is said to belong to optical branch. Acoustical branches pass through the origin whereas the optical branches do not.

ξ_1	ξ_2	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9
0.001	0.0	9.27E-4	8.032E-3	1.0000	1.0000	1.0010	1.0010	1.2100	1.2100	
0.100	0.0	0.05020	0.050200	0.0850	0.0850	0.9507	0.9507	1.0017	1.0017	
0.250	0.0	0.12550	0.125500	0.2130	0.2130	0.8760	0.8760	1.0081	1.0081	1.0082
0.500	0.0	0.25100	0.251000	0.4242	0.4242	0.7615	0.7615	1.0310	1.0310	1.0320
0.750	0.0	0.38000	0.380000	0.6260	0.6260	0.6350	0.6350	1.0531	1.0531	1.0670
1.000	0.0	0.50100	0.501000	0.8600	0.8600	1.1268	1.1268	1.1168	1.1168	1.1168
1.250	0.0	0.38000	0.380000	0.6260	0.6260	0.6350	0.6350	1.0531	1.0531	1.0670
1.500	0.0	0.25100	0.251000	0.4242	0.4242	0.7615	0.7615	1.0310	1.0310	1.0320
1.750	0.0	0.12550	0.125500	0.2130	0.2130	0.8760	0.8760	1.0081	1.0081	1.0082
1.900	0.0	0.05020	0.050200	0.0850	0.0850	0.9507	0.9507	1.0017	1.0017	
2.000	0.0	9.27E-4	8.032E-3	1.0000	1.0000	1.0010	1.0010	1.2100	1.2100	

Table 4.2 : Normalized Frequencies for an isotropic material
(Propagation in $x_1 - x_2$ plane)

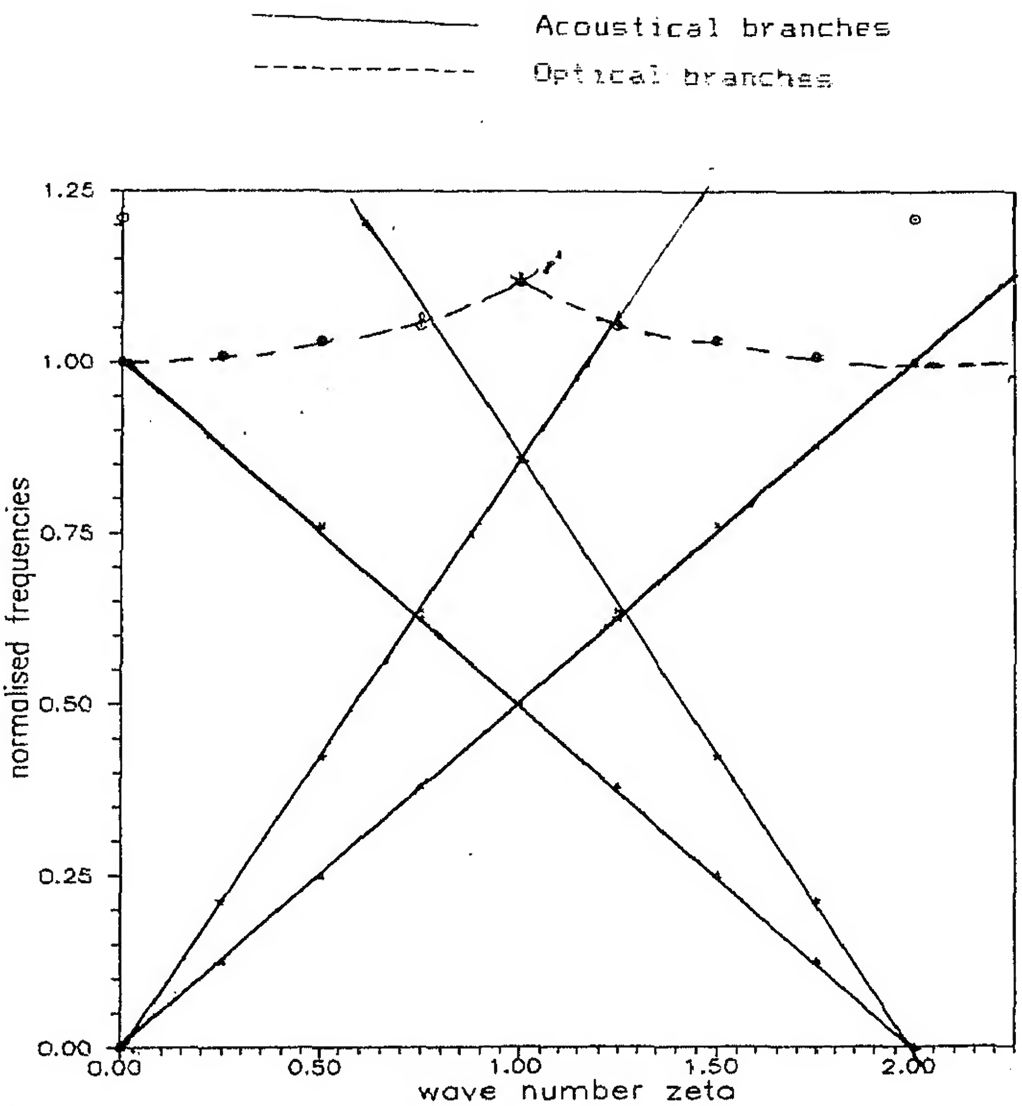


Fig. 4.3 Frequency Spectrum for an isotropic infinite medium for wave propagation in x_1 direction.

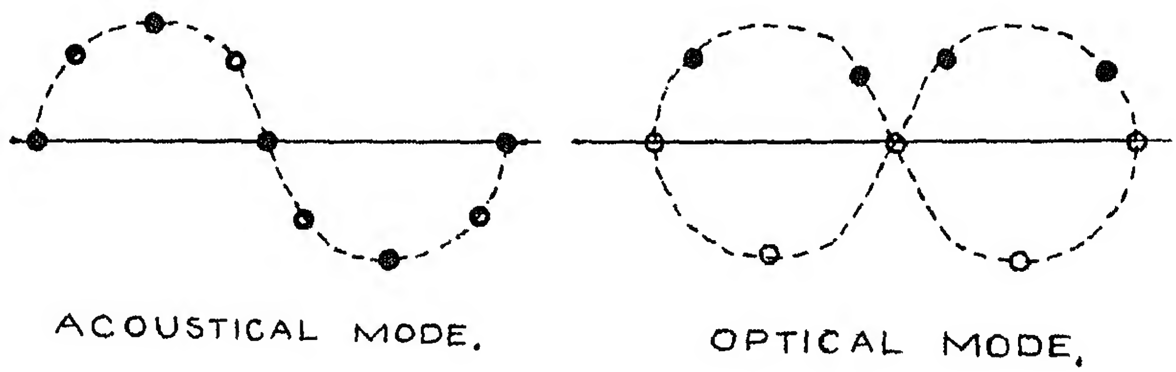


Fig. 4.4 Acoustical and optical modes of propagation.

3) The acoustical branches have the same slope ($d\omega$ / $d\zeta$) indicating that the group velocity is constant. This confirms the non dispersive effect of the isotropic medium.

4) The optical branches in contrast, are having different group velocities for different ζ implying dispersive phenomena.

It may be pointed out that the lowest branch corresponds to the shear wave propagation and the next lower one to the longitudinal wave in the medium.

4.5 RESULTS-COMPOSITE MATERIALS :

Two types of fiber reinforced composites are considered for analysis. Table 4.1 gives the material properties for the same. For the first one composite material the ratio E_f / E_m = 3.275 and ρ_f / ρ_m=4.

For the second material the ratios respectively are 20.1 and 2.1 respectively which corresponds to Glass/Epoxy composite. Ten lowest eigenvalues and corresponding eigen vectors are obtained for various values of ζ_1 and ζ_2 varying from 0 to 2.

Initially attempt were made to obtain the results using tolerance of 10^{-9}(Eqn. 4.1). Results showed inconsistancies and were not accurate. Hence tolerance has been increased to 10^{-6} which necessitated the use of double precision arithmetic. With these the results showed consistency and also the convergence of the eigen values improved appreciably.

Fig.4.5 shows rate of convergence of the eigen values with the eigenvalues. From the figure 4.5 it is evident that eigenvalues converged quickly and after about 40 iterations there

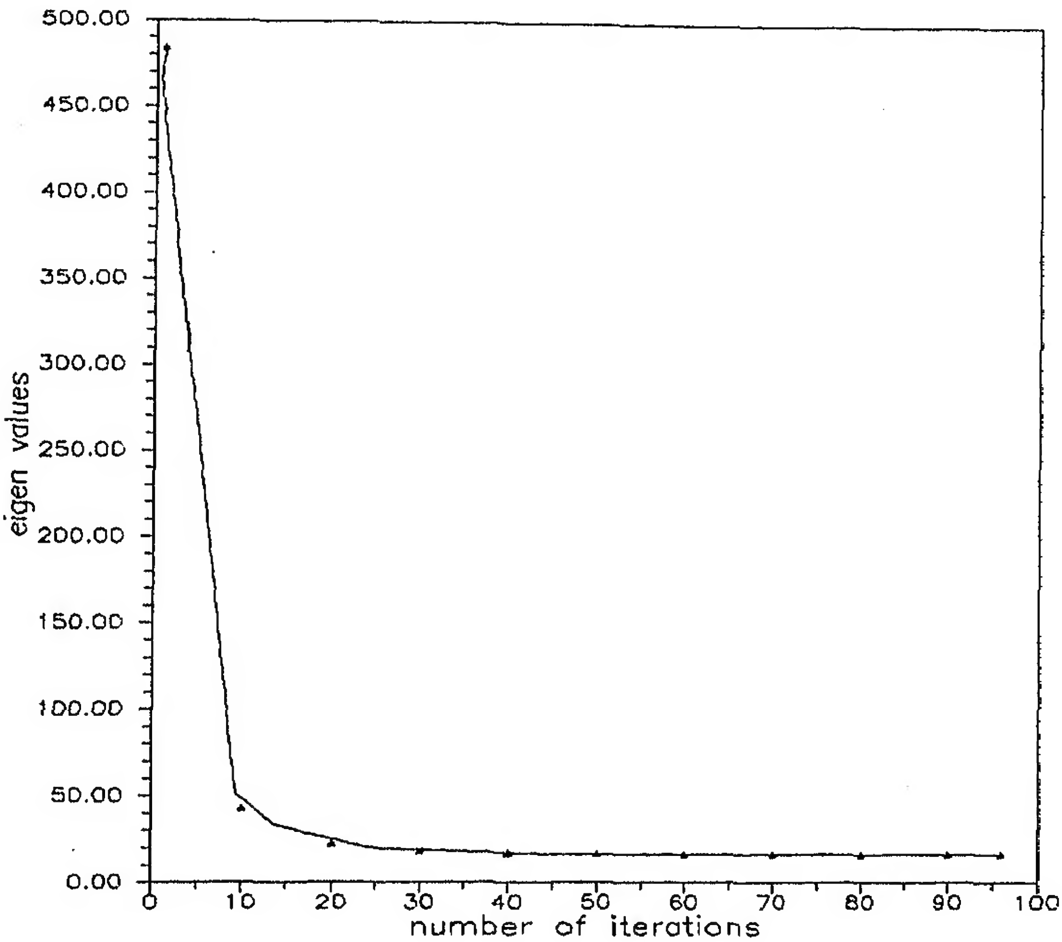


Fig. 4.5 Convergence rate of eigen values with number of jacobi iterations, for λ_9.

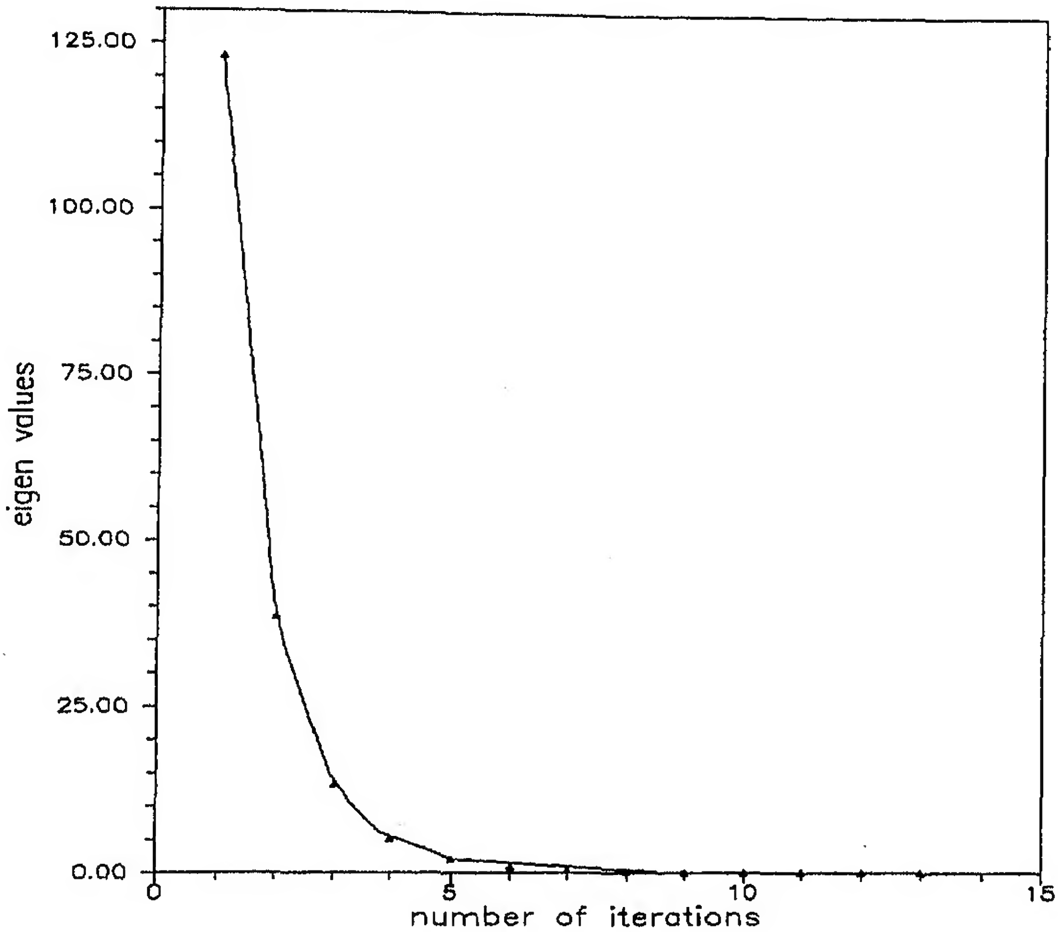


Fig. 4.5 Convergence rate of eigen values with number of jacobi iterations, for λ_1.

is very little defference between the values in subsequent iterations. Hence increasing the number of iterations has little effect. Hence in the present analysis the .o of iterations was limited to 45.

It also appeared that convergence of eigen values depends upon the range of elements in stiffness and mass matrices. Because of different material properties in different elements within the cell the stiffness and mass matrices tend to become ill conditioned. This results in increase of the number of iterations and use of double precision for composite materials.

4.5.1 COMPOSITE MATERIAL- 1 :

Results for the composite material with E_f / E_m = 3.275 is presented in figures 4.6a,b,c,d and tables 4.3,4.4,4.5,4.6 and 4.9.

4.5.1.1 PROPAGATION ALONG THE FIBERS :

Fig 4.6a and table 4.3 gives the dispersion relations for the present case. Comparison of these results with those of isotropic material (Fig. 4.3) shows the_following:

1) Acoustical branches for this material is almost same as to that of the isotropic material up to ζ_1= Ø.75. When ζ_1 approaches unity slight dispersion is evident.

2) Optical branches indicate larger dispersion.

3) In composites also there is symmetry in frequency spectrum about ζ_1= 1.Ø

ξ_1	ξ_2	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9
0.00	0.00	0.0008	0.1317	0.1718	0.3116	0.9968	0.9978	1.0000	1.0000	1.1023
0.10	0.00	0.0718	0.1395	0.1880	0.3257	0.9460	0.9467	1.0509	1.0512	1.1049
0.25	0.00	0.1535	0.1748	0.2734	0.3912	0.8680	0.8689	1.1167	1.1283	1.1314
0.50	0.00	0.2600	0.2664	0.4886	0.5635	0.7393	0.7408	1.1599	1.1926	1.2638
0.75	0.00	0.3718	0.3750	0.6128	0.6139	0.7195	0.7614	1.1415	1.1443	1.2395
1.00	0.00	0.4886	0.4913	0.4914	0.4926	0.9104	0.9209	0.9832	1.0063	1.3013
1.25	0.00	0.3718	0.3750	0.6128	0.6139	0.7195	0.7614	1.1415	1.1443	1.2395
1.50	0.00	0.2600	0.2664	0.4886	0.5635	0.7393	0.7408	1.1599	1.1926	1.2638
1.75	0.00	0.1535	0.1748	0.2734	0.3912	0.8680	0.8689	1.1167	1.1283	1.1314
1.90	0.00	0.0718	0.1395	0.1880	0.3257	0.9460	0.9467	1.0509	1.0512	1.1049
2.00	0.00	0.0008	0.1317	0.1718	0.3116	0.9968	0.9978	1.0000	1.0000	1.1023

Table 4.3 : Normalized frequency for Compisite material ($E_f/E_m = 3.275$) ,
Propagation along x_1 direction

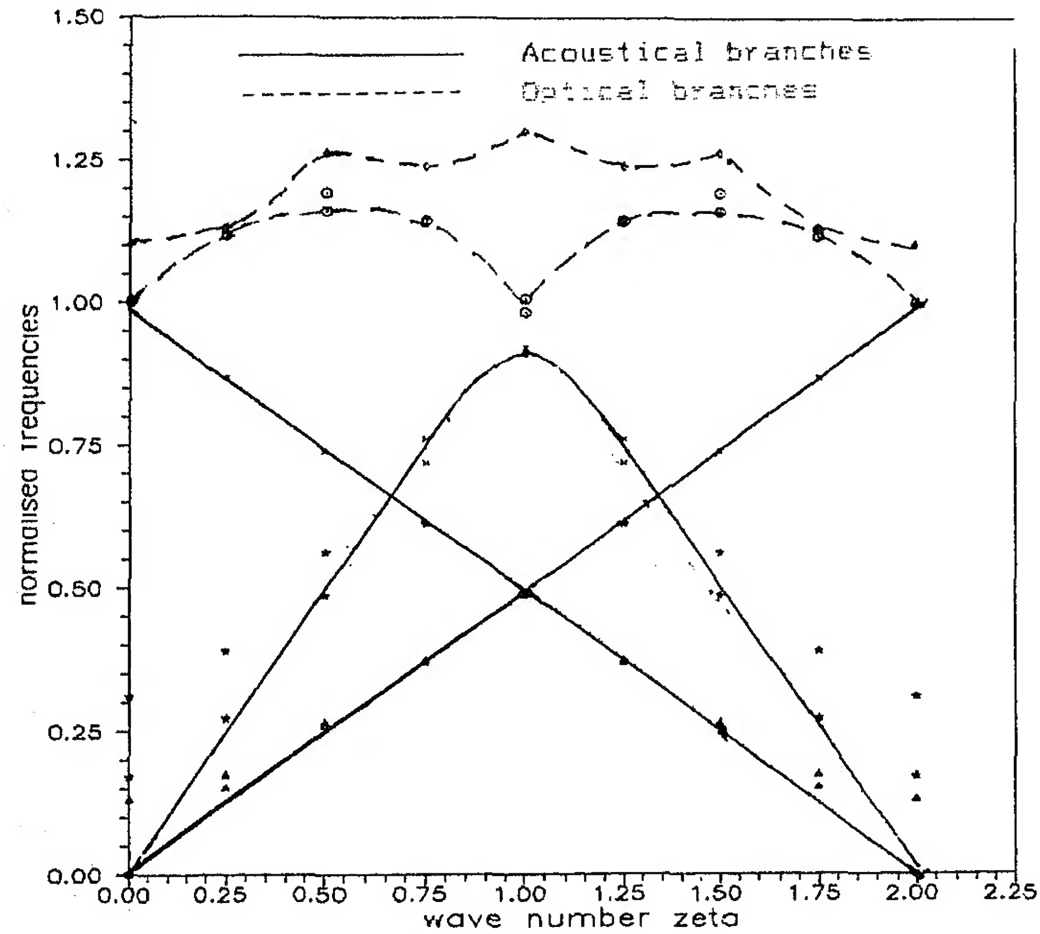


Fig. 4.6a Frequency spectrum for composite material (E_f/E_m = 3.275)
For wave propagating along the fibers.

4.5.1.2 PROPAGATION PERPENDICULAR TO FIBERS :

Two cases of propagation perpendicular to fibers are considered ;

a) In the plane of plate (X_1, X_2 plane)- plane stress case

b) In the thickness plane (X_2, X_3 plane) - plane strain case

Fig. 4.6(b) and Table 4.4 illustrate the former case while Fig 4.6d and Table 4.6 the second case.

From Fig.4.6(b) and Table 4.4 the following points may be noted.

1) Dispersive phenomenon is predominant even for comparatively small values of ζ_1 = 0.25. This is in contrast to the behavior in the direction of fibers (Fig.4.6). Even acoustical branches show considerable dispersion.

2) The slope of the branches $\frac{d\omega}{dk}$ is zero at ζ =1.0 (d = λ = 1.0) indicates the group velocity at ζ =1.0 is zero. This can be attributed to the scattering of waves as the wave length approach the size of unit cell.

3) Values of frequencies are appreciably less for propagation in ζ_2 direction as compared to those for ζ_1 direction. This can be attributed to the difference in stiffness of the medium in both the directions (anisotropy). As the composite is more stiff in fiber direction the values of frequencies are more for propagation along the fibers.

Fig.4.6(d) and Table 4.6 show the frequency spectrum for harmonic wave propagation in the x_2, x_3 plane.

Following observations can be made with reference to Fig. 4.6(d) and Table 4.6.

ξ_{q1}	ξ_{q2}	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9
0.00	0.00	0.0008	0.1317	0.1718	0.3116	0.9968	0.9978	1.0000	1.0000	1.1023
0.00	0.10	0.0633	0.1574	0.1838	0.3126	0.9973	0.9982	1.0002	1.0002	1.0990
0.00	0.25	0.1264	0.2483	0.2508	0.3176	0.9977	0.9980	1.0025	1.0030	1.0844
0.00	0.50	0.1747	0.3300	0.4090	0.4133	0.9887	0.9887	1.0115	1.0125	1.0546
0.00	0.75	0.1857	0.3441	0.5245	0.5264	0.9478	0.9479	1.0196	1.0231	1.0433
0.00	1.00	0.1762	0.3492	0.5561	0.5576	0.8915	0.8927	1.0261	1.0267	1.0451
0.00	1.25	0.1857	0.3441	0.5245	0.5264	0.9478	0.9479	1.0196	1.0231	1.0433
0.00	1.50	0.1747	0.3300	0.4090	0.4133	0.9887	0.9887	1.0115	1.0125	1.0546
0.00	1.75	0.1264	0.2483	0.2508	0.3176	0.9977	0.9980	1.0025	1.0030	1.0844
0.00	1.90	0.0633	0.1574	0.1838	0.3126	0.9973	0.9982	1.0002	1.0002	1.0990
0.00	2.00	0.0008	0.1317	0.1718	0.3116	0.9968	0.9978	1.0000	1.0000	1.1023

Table 4.4 : Normalized frequency for Composite material ($E_f / E_m = 3.275$), propagation along x_2 direction

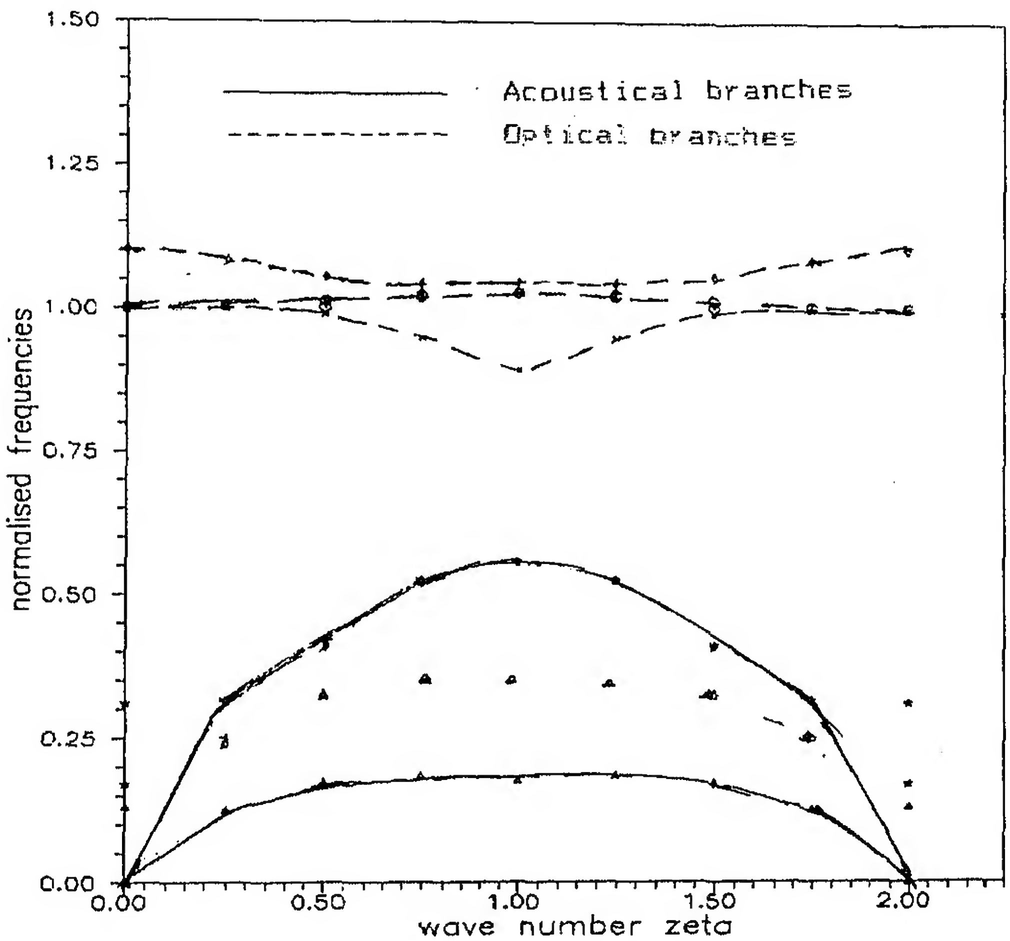


Fig. 4.6b. Frequency spectrum for composite material (E_f/E_m = 3.275) for wave propagating perpendicular to the fibers but in the same plane of fibers.

0.00	0.00	0.0044	0.3131	0.3131	0.3142	0.5477	0.5480	1.0000	1.3004	1.3040
0.10	0.00	0.0746	0.3147	0.3186	0.3233	0.5469	0.5472	0.9995	1.2985	1.3017
0.25	0.00	0.1706	0.3205	0.3550	0.3729	0.5420	0.5435	0.9979	1.2892	1.3060
0.50	0.00	0.2664	0.3414	0.4997	0.5201	0.5257	0.5282	1.0038	1.2653	1.3008
0.75	0.00	0.3420	0.3737	0.4815	0.5008	0.6665	0.6690	1.0464	1.2479	1.2865
1.00	0.00	0.3926	0.3957	0.4485	0.4805	0.7341	0.7341	1.1007	1.2440	1.2818
1.25	0.00	0.3420	0.3737	0.4815	0.5008	0.6665	0.6690	1.0464	1.2479	1.2865
1.50	0.00	0.2664	0.3414	0.4997	0.5201	0.5257	0.5282	1.0038	1.2653	1.3008
1.75	0.00	0.1706	0.3205	0.3550	0.3729	0.5420	0.5435	0.9979	1.2892	1.3060
2.00	0.00	0.0044	0.3131	0.3131	0.3142	0.5477	0.5480	1.0000	1.3004	1.3040

Table 4.6 : Normalized frequencies for Composite material(E_f/E_m = 3.275), propagation in x_2-x_3 plane along x_2 direction

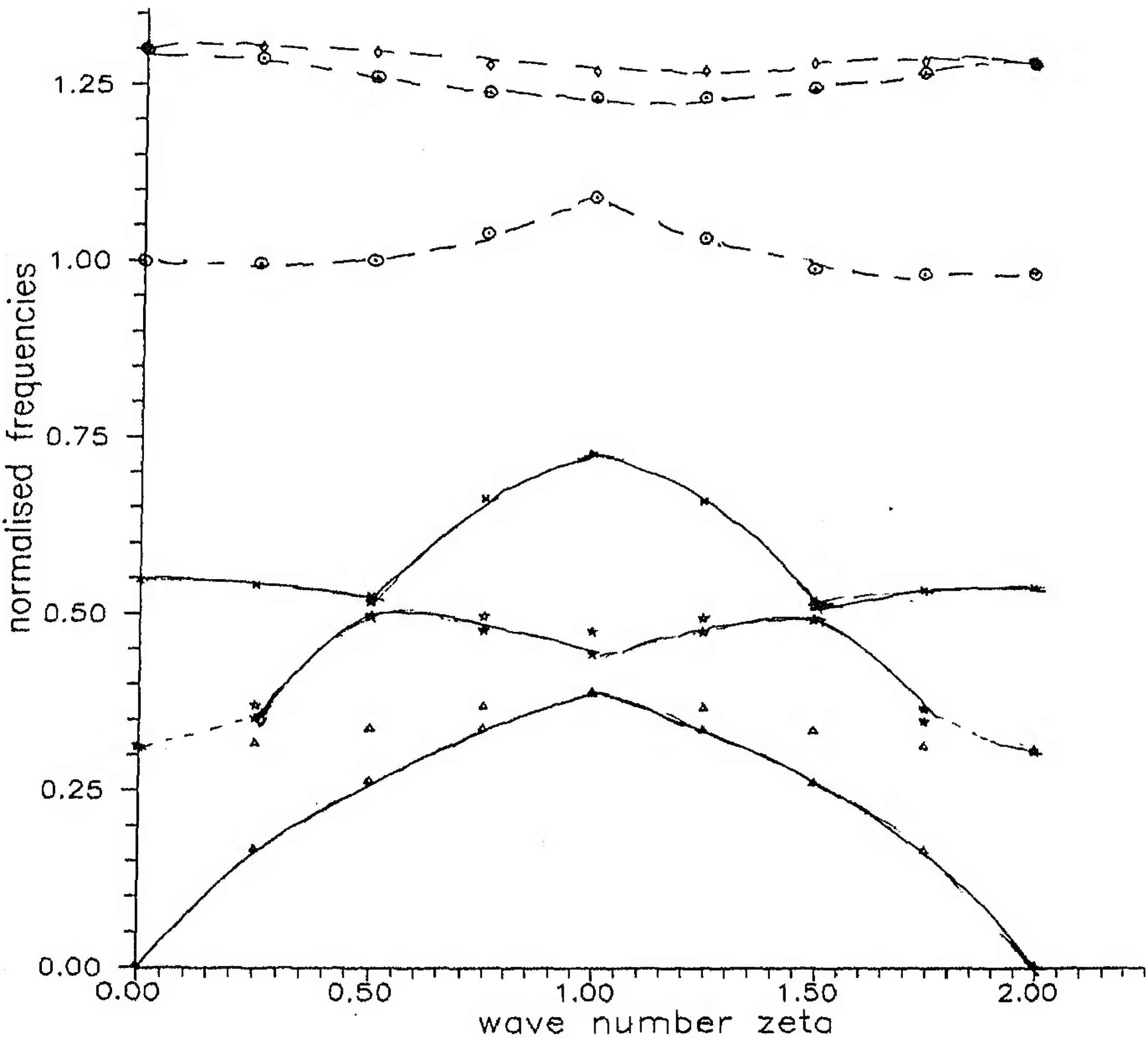


Fig.4.6d Freqency spectrum for wave propagation in composite material($E_f / E_m = 3.275$), propogation in $x_2 - x_3$ plane along x_2 direction.

1) There are some differences between the frequency spectra for wave propagation in plane stress and plane strain cases even if the direction of propagation is perpendicular to fibers. The difference between the first and second acoustical branches at ξ = 1.Ø is large for plane stress as compared to plane strain case. This is because the lowest frequency at ξ = 1.Ø for plane strain case is nearly twice that of the lowest frequency in plane stress case but the next mode value are appreciably not different.

2) There is a additional optical branch appearing in this case.

3) In this case also the group velocity is zero at ξ = 1.Ø

4) There is a distinct change in the slope of the branches at ξ=Ø.5

4.5.1.3 PROPAGATION DIRECTION ORIENTED AT 45° TO FIBERS :

Fig. 4.7(c) and Table 4.5 show the frequency spectrum for the waves propagating at 45° ($\xi_1 = \xi_2$) to the fibers and in the same plane The following observations can be made.

1) In the case of acoustical branches there is little difference between longitudinal and shear waves up to ξ = Ø.25. Then the difference increases with further increase in ξ.

2) Slope of the acoustical branches is zero at ξ =Ø.75 indicating that the group velocity is zero at that point.

3) In the optical branches the behavior is more complex. Some branches indicate that the group velocities at ξ =Ø.25, ξ =Ø.5 , ξ = Ø.75 and 1 are different where as some have zero group velocity at ξ = Ø.25, Ø.5, Ø.75

ξ_1	ξ_2	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9
0.00	0.00	0.0008	0.1317	0.1718	0.3116	0.9968	0.9978	1.0000	1.0000	1.1023
0.25	0.25	0.2574	0.2644	0.2875	0.4058	0.8764	0.8774	1.0947	1.2997	1.3774
0.50	0.50	0.3951	0.4118	0.5323	0.5892	0.7613	0.7622	1.0949	1.1677	1.2417
0.75	0.75	0.4932	0.5019	0.6620	0.6712	0.7295	0.7546	1.0538	1.0911	1.1595
1.00	1.00	0.4978	0.5002	0.6368	0.6412	0.8477	0.8773	0.9210	0.9617	1.0739
1.25	1.25	0.4932	0.5019	0.6620	0.6712	0.7295	0.7546	1.0538	1.0911	1.1595
1.50	1.50	0.3951	0.4118	0.5323	0.5892	0.7613	0.7622	1.0949	1.1677	1.2417
1.75	1.75	0.2574	0.2644	0.2875	0.4058	0.8764	0.8774	1.0947	1.2997	1.3774
2.00	2.00	0.0008	0.1317	0.1718	0.3116	0.9968	0.9978	1.0000	1.0000	1.1023

Table 4.5 : Normalized frequencies for Composite material (E_f/E_m = 3 . 275), propagation along 45° to x_1 direction

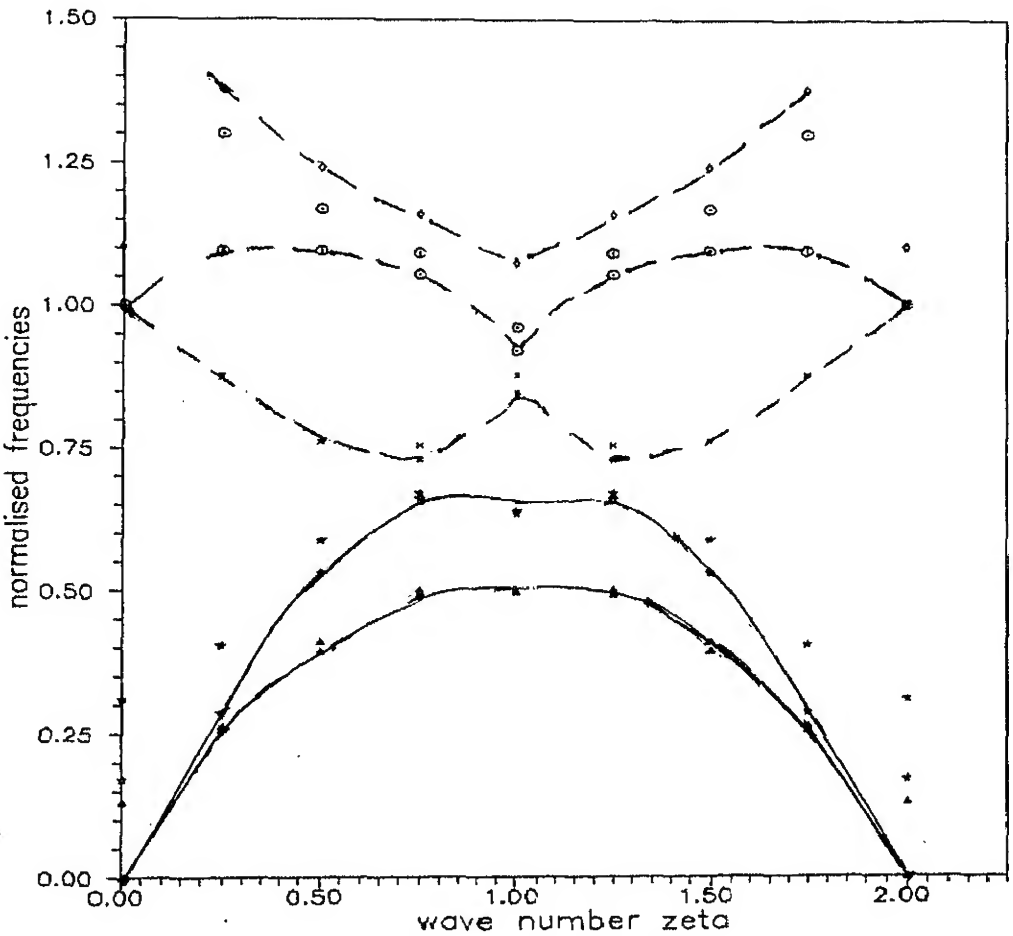


Fig. 4.6c. Wave propagating with $\xi_1 = \xi_2$ (at 45° to the x_1- axis), E_f/E_m = 20.1.

4) When compared to frequency spectrum for propagation along and perpendicular to fibers (Figures 4.6 (a),(b)) The absolute values of eigen values for lower two modes of circular waves ($\xi_1 = \xi_2$) have intermediate values between those for propagation along and perpendicular to the fiber direction.

4.5.2 GLASS EPOXY COMPOSITE MEDIUM :

A glass fiber reinforced composite material was also investigated. Figs. 4.7 (a,b) and Tables 4.7 and 4.8 present the results for wave propagating in this medium. Two cases considered here are propagation along the fibers and 45° to the same.

4.5.2.1 PROPAGATION ALONG THE FIBERS :

Fig. 4.7(a) and Table 4.7 gives the frequency spectrum for the glass fiber composites for propagation along the fibers. From the graph the following points can be observed.

1) For the lowest acoustical branch for the corresponding shear waves there is no dispersive effect till ξ_1 = Ø.75 and further increase in ξ the group velocity gradually decreases to zero at ξ = 1.Ø.

2) For the second acoustical branch corresponding to longitudinal wave there is a perceptible change in group velocity at ξ = Ø.25 itself then gradually reduces to zero at ξ = 1.Ø

3) When compared to acoustical branches the optical branches show more complex pattern. But for all the optical branches also the group velocity is zero at ξ = 1.Ø.

4) All the optical branches show distinct change in group

ξ_1	ξ_2	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9
0.00	0.00	0.0042	0.5172	0.5527	0.6339	0.9316	1.0098	1.0000	1.0103	1.0103
0.25	0.00	0.2549	0.8650	0.8775	0.8886	1.0626	1.0635	1.1044	1.2341	1.2344
0.50	0.00	0.4919	0.8747	0.8987	0.9153	0.9972	0.9987	1.1184	1.2802	1.3011
0.75	0.00	0.6899	0.8909	0.9373	0.9557	0.9581	0.9594	1.1565	1.2472	1.2773
1.00	0.00	0.7787	0.9151	0.9151	0.9152	0.9532	0.9613	1.1912	1.2153	1.2217
1.25	0.00	0.6899	0.8909	0.9373	0.9557	0.9581	0.9594	1.1565	1.2472	1.2773
1.50	0.00	0.4919	0.8747	0.8987	0.9153	0.9972	0.9987	1.1184	1.2802	1.3011
1.75	0.00	0.2549	0.8650	0.8775	0.8886	1.0626	1.0635	1.1044	1.2341	1.2344
2.00	0.00	0.0042	0.5172	0.5527	0.6339	0.9316	1.0098	1.0000	1.0103	1.0103

Table 4.7 : Normalized frequency for Glass-epoxy Composite material(E_f/E_m = 20.1), propagation in $x_1 - x_2$ plane, along x_1 direction

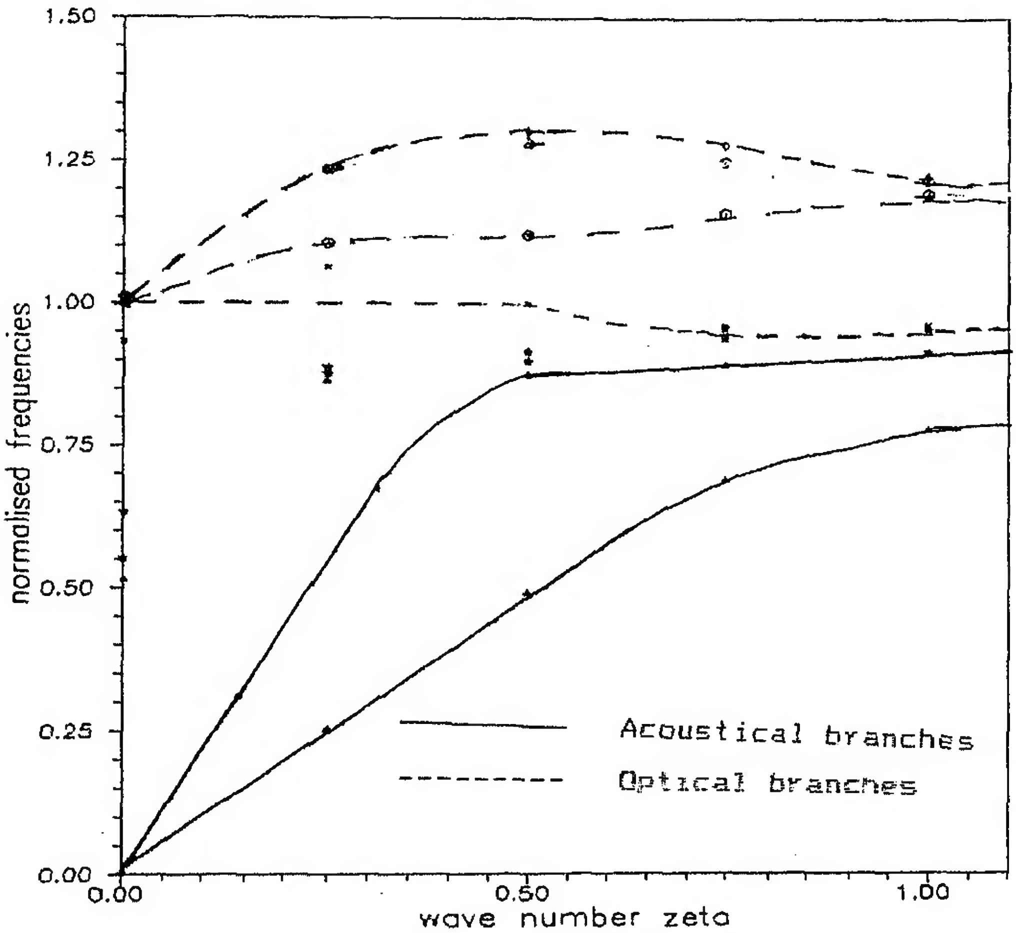


FIG Fig. 4.7a. Frequency spectrum for glass - epoxy composite medium ($E_f/E_m = 20.1$) For wave propagating along the fibers.

velocity at $\xi = 0.5$

4.5.2.2. PROPAGATION OF CIRCUALR WAVES:

For propagation of circular wave ($\xi_1 = \xi_2$) the frequency spectrum is presented in Fig. 4.7b and Table 4.8. From the propagation of waves at 45° to the fiber orientation appears to be similar to that of the propagation along the fibers.

ξ_{f1}	ξ_{f2}	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9
0.25	0.50	0.2767	0.3660	0.4309	0.4609	0.8850	0.8858	1.0638	1.1185	1.1300
0.25	0.75	0.2788	0.4003	0.5283	0.5358	0.8813	0.8818	1.0637	1.0871	1.0990
0.25	1.00	0.2740	0.4152	0.5423	0.5451	0.8409	0.8421	1.0681	1.0697	1.0955
0.50	0.25	0.3215	0.3219	0.4969	0.5716	0.7449	0.7464	1.1213	1.1830	1.2607
0.50	0.75	0.4268	0.4714	0.5552	0.5953	0.7708	0.7710	1.1025	1.1507	1.1624
0.50	1.00	0.4456	0.5021	0.5233	0.5731	0.7550	0.7554	1.0887	1.1196	1.1660
0.75	0.25	0.4089	0.4093	0.6210	0.6238	0.7265	0.7641	1.1317	1.1361	1.1976
0.75	0.50	0.4651	0.4688	0.6446	0.6500	0.7370	0.7646	1.1008	1.1172	1.1861
0.75	1.00	0.4910	0.4948	0.6343	0.6633	0.7055	0.7449	1.0312	1.0562	1.0909
1.00	0.25	0.5073	0.5103	0.5170	0.5190	0.9164	0.9238	0.9825	1.0088	1.2875
1.00	0.50	0.5388	0.5398	0.5620	0.5683	0.9085	0.9120	0.9614	0.9958	1.2898
1.00	0.75	0.5343	0.5377	0.6136	0.6193	0.8825	0.8921	0.9334	0.9756	1.2196

Table 4.9 : Normalized frequencies for Composite material($E_f/E_m = 3.275$), for unequal values of ξ_{f1} and ξ_{f2}

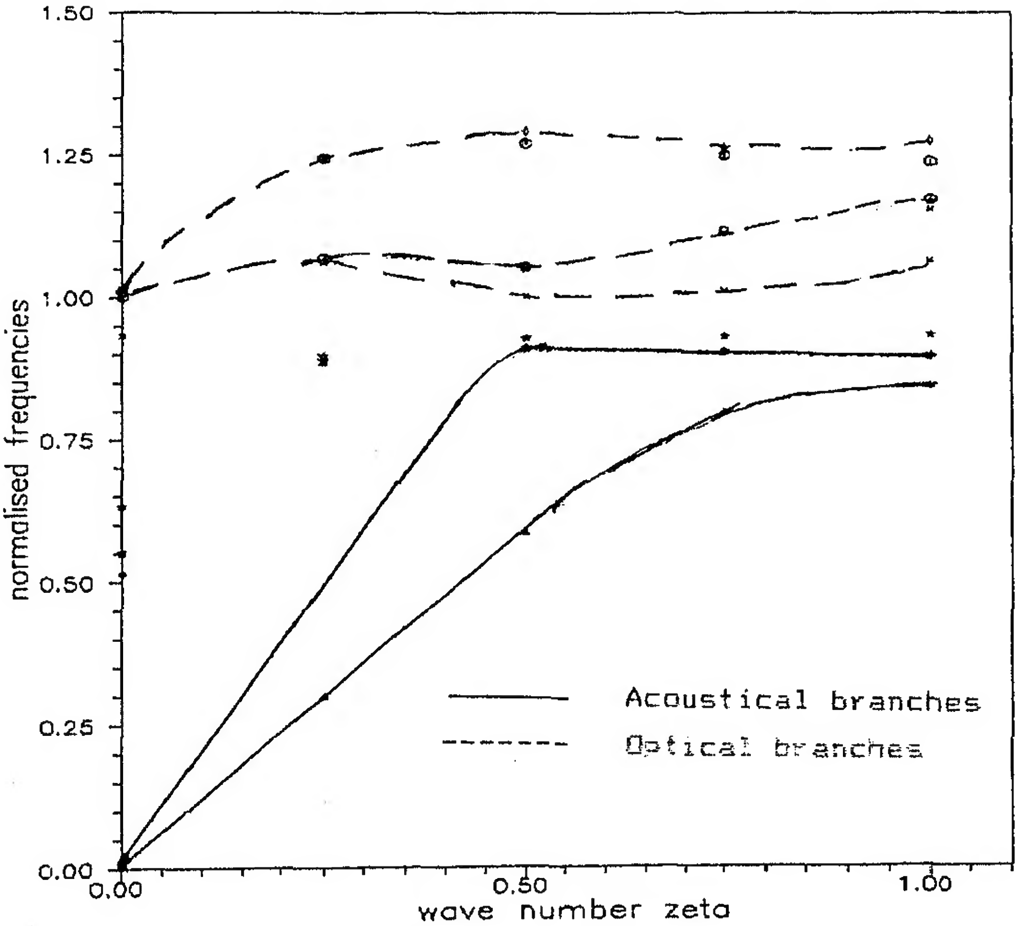


Fig. 4.7b. For wave propagating at 45° to x_1 direction ($\xi_1 = \xi_2 = \xi$) (E_f/E_m=20.1).

CHAPTER 5

CONCLUSIONS

Finite element analysis of harmonic wave propagation studies in composites materials was carried out for two dimensional wave propagation. Concepts of unit cells and Floquet theory were used in modelling the behavior of the medium. he eigen value problem obtained by finite element formulation was solved for ten lowest eigen frequencies as a function of wave number ζ. The frequency spectrum (dispersion relations) were obtained for various cases of propagation in the plane of plate and in thickness direction with values of ζ varying from Ø- 2.Ø .

Following conclusions can be drawn from the analysis carried out .

1) The behavior of composite materials for harmonic wave propagation is quite complex when compared to that of isotropic materials.

2) The ratio of stiffness of individual constituents in the medium is an important factor affecting the dispersion phenomena.

3) The dispersion phenomena in a composite medium is dependent on the mode of propagation and the wavelength /frequency of propagation. As the half wavelength approaches the dimension of the unit cell, dispersion is pronounced and at $d = \frac{\lambda}{2}$ the group velocity is zero indicting scattering of waves within the medium.

4) There is symmetry of frequency spectrum about $\zeta = 1$. This may be attributed to the fact that fiber and matrix materials are symmetrically situated about the centre of the unit cell.

5) The dispersion effect depends on the direction of propagation. The dispersion effect is pronounced in the direction perpendicular to the fibers .

6) The longitudinal and shear waves are comparatively less affected by the dispersive nature of the medium than the higher modes.

7) Finite element method can be a suitable technique to study the wave propagation in composite materials.

BIBLIOGRAPHY

1. Zienkiewicz,O.C.," The Finite Element Method " , McGraw-Hill Book Company, Third Edition 1977.

2. Osaki, T and Kimpara I , : "Evaluation of Defects in Unidirectional Fiber Composite by Elastic Wave Propagation", Proc. of the Fourth Conf. on Composite Materials, June 27 - 29, 1988.

3. Oshima T.etal.:"Stress Wave Propagation in a Rectangula Composite Beam" ,Proc.of the Fourth Japan -US Conf.on CompositeMaterials,June27-29,1988.

4. Navi,P.,"Waves and Vibrations in Elastic Fibre - Composite Medium and Plates ", Vibrations and Stress Analysis ,pp. 135 - 143.

5. Kohn W, J.A. Krumhansl and E.H. Lee : "Variational Methods for Dispersion Relations and Elastic Properties in Composite Materials", Trans. ASME, Jl. Applied Mech., Vol. 39, June 1972, pp. 327 - 336.

6. Dong, S.B. and Nelson R B, : "On Natural Vibrations and Waves in Laminated Orthotropic Plates", Trans. ASME, Jl. Applied Mech., Vol. 239, Sept. 1972, pp. 739 - 745.

7. Yang, W H and Lee E.H. : "Modal Analysis of Floquet Waves in Composite Materials ", Trans. ASME, Jl. App. Mech., June 1974, pp. 429 - 433.

8. Richard B. Nelson and Parviz Navi : "Harmonic Wave Propagation in Composite Materials", J. *of* Acoust. Soc. *of* Am. , Vol. 57 , No.4 , April 1975 , pp. 773-781.

9. Golub, G.H., Jemming L. Yang W.H., " Waves in Periodic Structured Media", J.Of Comp. Phy., Vol 17, pp 349-357 1975

10. Minagawa, S. et al. : "Finite Element Analysis of Harmonic Waves in Layered and Fiber Reinforced Composites", Int. Jl. for Num. Methods in Engg., Vol.17, 1981, pp. 1335 - 1353.

11. Minagawa, S.et al. : "Finite Element Analysis of Harmonic Waves in Layered and Fiber Reinforced Composites", J. Comp.

Struc., Vol 19 (1-2), 1984, pp 119-28

12. Datta, S.K., et al. : "Wave propagation in laminated composite plates" , Jl. Acoust.Soc of America. No. 83 (6) June 1988

13. Mal, A.K. and Y. Barcohen : "Stress Waves in Layered Composite Laminates", Proc. of the Fourth Japan - US Conference on Composite Materials, June, pp. 27 - 29, 1988.

14. ToshiukiOshima, et al., " Stress Waves in a Rectangular Composite Beam " , Proc., Forth Japan U.S., conf Comp. Mat., Jan 27-29 1986

15. Masanory Koshiba " Finite Element Analysis of Wave Scattring in an Elastic Plate Wave Gaide ", IEEE Trans Sonic and Ultasonicsa Vol Su 31 No. 1 Jan 1984

16. Joseph L. Rose : "Ultrasonic Wave Propagation Principles in Composite Material Inspection ", Materials Evaluation / 43 / April 1985, pp. 481 - 483.

17. Rose, J.L , A.S.D. Wang and E.W. Deska : "Wave Profile Analysis in a Unidirectional Graphite Epoxy Plate", J. of Composite Materials, Vol. 8, Oct., 1974, pp. 419 - 425.

18. Tauchert, T.R. and Gazelsu : "An Experimental Study of Dispersion of Stress Wave in a Fiber Reinforced Composite", Trans. ASME, J. of App. Mech., Vol. 79, Mar., 1972, pp. 95 - 102.

19. Munson, D.E. and K.W. Schuler : "Steady Wave Analysis of Wave Propagation in Laminar and Mechanical Mixtures", J. of Composite Materials, Vol. 5, July 1971, pp. 286.

20. Hemann and George Y. Baaklini "The Effect of Stress on Ultrasonic Pulses in Fiber Reinforced Composites", SAMPE J., July / Aug. 1986, pp. 9.

21. Teti, R and G. Caparino : "NDE of THick GFRP Composites through Ultrasonic Wave Form Detection", Developments in the Science and Technology of Composite Materials, ECCM -' 3, 3rd European Conference on Composite Materials, 20 - 23 ,March, 1989, Bourdeaux, France, pp. 793 - 800.

22. Kinra, V.K. and V. Dayal : "A New Technique for Ultrasonic -

Non destructive Evaluation of Thin Specimens", Experimental Mechanics, Vol. 28, No. 3, Sept. 1988, pp. 288 - 297.

23. Navadunsky, J.J. and Lucas,J.J., : " Early Fatigue Damage Location In Composite Materials", J. Comp. Mat. Vol 9 Oct 1975 pp 394-408

24. Rose, J.L., and Carsonm.,: " Ultasonic Nondestructive Test Procedure For The Early Detection of Fatigue Damage and The Prediction of Remaing Life " , Mat . Eval. , Apr 1980 pp 27-34

25. Jones, R.M. : "Mechanics of Composite Materials", 1975.

26. Pei ,Chi Chen Wang A.S.D., " COntrol Volume Analysis of Eleastic Wave Front in Composite Materials", J.Comp. Mat Vol 4 Oct 1970 pp 444-461

27. Tewary, V.K. : "Mechanics of Fiber Composites", Wiley Eastern Ltd., 1978.

28. Lawrence J .Broutman and Richard H.Krock :" Composite Materials " , Vol.2, " Mechanics of Composite Materials"Edited by Sendeckyj , G.P., Academic Press, NY, 1974.

29. Kittlel.,C., "Intoduction To Solid State Physics " Willey Eastern Edition, Newyork, 3rd Edition 1968 .

30. Kolsky, H. : "Stress Waves in Solids", Dover Publishers Inc. NY, 1963.

31. Humar, J.L. : "Dynamics of Structures", Prentice Hall Ltd., 1989.

32. Chueng.,Y.K., and Yeo.,M.P., " A Practical Introduction To Finite Element Analysis", Pitman Publishing Ltd, 1979

33. Cook, R.D. : "Concepts and Applications of Finite Element Analysis", John Wiley and Sons., 1981.

34. Wilkinson, " The Algebriac Eigen Value Problem ", Oxford: The Clerendon Press 1976

35. Sastry, J.V. "In PLane Vibration of Layered Rings on Periodic Radial Supports Using Finite Element Method " M.Tech Thesis Dept. Mech Engg., I.I.T., Kanpur,1986

36. Bathe, K.J. and E.L. Wilson : "Numerical Methods in Finite Element Analysis",PHI Pvt. Ltd., 1987.

37. Reddy, J.N. : " An Introduction to the Finite Element Analysis ", 1986.

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